Mathematics

Question1

Let $f : R \rightarrow R$ be defined by $f(x) = 5^{-|x|} + \operatorname{sgn}\left(5^{-x}\right)$, where sgn x denotes signum function of x. Then f is

Options:

A.

One-one but not onto

B.

Onto but not one-one

C.

Both one-one and onto

D.

Neither one-one nor onto

Answer: D

Solution:

Given, $f(x) = 5^{|x|} + \operatorname{Sgn}\left(5^{x}\right)$

$\because \quad 5^{x} > 0 \forall x \in R$

$\therefore \quad \operatorname{sgn}\left(5^{x}\right) = 1$

And $\quad |x| \geq 0 \Rightarrow -|x| \leq 0$

$\Rightarrow \quad 5^{x} \leq 5^{0} \Rightarrow 5^{x} \leq 1$

$\quad 5^{|x|} \leq 5^{0} \Rightarrow 5^{|x|} \leq 1$

$\therefore \quad f(x) = 5^{|x|} + 1 \leq 1 + 1 \leq 2$

$\because \quad f(1) = f(-1)$

$\therefore f(x)$ is neither one-one nor onto.

Question2

If the range of the real valued function $f(x)=\frac{x^2+x+k}{x^2-x+k}$ is $\left[\frac{1}{3}, 3\right]$, then $k=$

Options:

A.

-2

B.

-1

C.

1

D.

2

Answer: C

Solution:

$f(x)=\frac{x^2+x+k}{x^2-x+k}$

$\because$ Range $=\left[\frac{1}{3}, 3\right]$

For range let $f(x)=y$

$$y=\frac{x^2+x+k}{x^2-x+k}$$
$$\Rightarrow \quad x^2(y-1)-x(y+1)+ky-k=0$$
$$\therefore \quad D \geq 0$$
$$\Rightarrow \quad (y+1)^2-4k(y-1)(y-1) \geq 0$$
$$\Rightarrow \quad (y+1)^2-(2\sqrt{k}(y-1))^2 \geq 0$$
$$\Rightarrow \quad (y+1+2\sqrt{k}(y-1))$$
$$(y+1-2\sqrt{k}(y-1) \geq 0$$
$$\Rightarrow \quad (y(2\sqrt{k}+1)+1-2\sqrt{k})$$
$$(y(2\sqrt{k}-1)-2\sqrt{k}-1) \leq 0 \qquad \ldots(i)$$
$$\because \quad y \in \left[\frac{1}{3}, 3\right]$$
$$(3y-1)(y-3) \leq 0 \qquad \ldots(ii)$$

Comparing Eqs. (i) and (ii),

$2\sqrt{k}+1=3 \Rightarrow 2\sqrt{k}=2 \Rightarrow k=1$

Question3

The value of the greatest integer k satisfying the inequation $2^{n+4}+12 \geq k(n+4)$ for all $n \in N$ is

Options:

A.

7

B.

8

C.

9

D.

10

Answer: B

Solution:

$$
\begin{aligned}
& 2^{n+4}+12 \geq k(n+4) \\
& \quad k \leq \frac{2^{n+4}+12}{n+4} \\
& \text{Let } n+4=t \\
& \Rightarrow \quad k \leq \frac{2^t+12}{t}, t \geq 5 \\
& \quad t=5, k \leq \frac{2^5+12}{5}=8.81 \\
& \quad t=6, k \leq \frac{2^6+12}{6}=1267 \\
& \quad t=7, k \leq \frac{2^7+12}{7}=20 \\
& \because \quad t=5 \Rightarrow k \text{ will be minimum } =8.8 \\
& \therefore \quad [k]=8
\end{aligned}
$$

Question4

If the system of simultaneous linear equations $x-2y+z=0, 2x+3y+z=6$ and $x+2y+pz=q$ has infinitely many solutions, then

Options:

A.

$p+q=4$

B.

$pq=\frac{48}{49}$

C.

$q-p=3$

D.

$\frac{p}{q}=4$

Answer: C

Solution:

$$\begin{aligned} x-2y+z&=0\\ 2x+3y+z&=6\\ x+2y+pz&=q \end{aligned}$$

System has infinite solution,

$$\therefore \quad \Delta=\Delta_x=\Delta_y=\Delta_z=0$$

$$\Delta=\begin{vmatrix}1 & -2 & 1\\ 2 & 3 & 1\\ 1 & 2 & p\end{vmatrix}=0$$

$$\Rightarrow \quad 1(3p-2)+2(2p-1)+1(4-3)=0$$

$$\Rightarrow \quad 3p-2+4p-2+1=0$$

$$\Rightarrow \quad 7p=3 \Rightarrow p=\frac{3}{7}$$

$$\Delta_z=\begin{vmatrix}1 & -2 & 0\\ 2 & 3 & 6\\ 1 & 2 & q\end{vmatrix}=0$$

$$\Rightarrow \quad 1(3q-12)+2(2q-6)=0$$

$$\Rightarrow \quad 3q-12+4q-12=0$$

$$\Rightarrow \quad 7q=24 \Rightarrow q=\frac{24}{7}$$

$$\therefore \quad q-p=\frac{24}{7}-\frac{3}{7}=3$$

Question5

If the system of linear equations $(\sin\theta)x-y+z=0$, $x-(\cos\theta)y+z=0, x+y+(\sin\theta)z=0$ has non-trivial solution, then the least positive value of θ is

Options:

A.

$\frac{\pi}{6}$

B.

$\frac{\pi}{4}$

C.

$\frac{\pi}{3}$

D.

$\frac{\pi}{2}$

Answer: D

Solution:

We have,

$(\sin\theta)x - y + z = 0$
$x - (\cos\theta)y + z = 0$
$x + y + (\sin\theta)z = 0$

System has non-trivial solution.

$\therefore \quad \Delta = 0$

$$\begin{vmatrix} \sin\theta & -1 & 1 \\ 1 & -\cos\theta & 1 \\ 1 & 1 & \sin\theta \end{vmatrix} = 0$$

$\Rightarrow \quad \sin\theta(-\sin\theta\cos\theta - 1) + 1(\sin\theta - 1) + 1(1 + \cos\theta) = 0$

$\Rightarrow \quad -\sin^2\theta\cos\theta - \sin\theta + \sin\theta - 1 + 1 + \cos\theta = 0$

$\Rightarrow \quad \cos\theta\left(1 - \sin^2\theta\right) = 0$

$\Rightarrow \quad \cos^3\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

Question6

. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 2 \\ 2 & 4 & 2 \end{bmatrix}$, then $\sqrt{|\operatorname{adj}(AB)|} =$

Options:

A. 176

B. 208

C. 198

D. 234

Answer: C

Solution:

We have,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix}$$

$$|A| = 1(1-3) - 2(2-1) + 3(6-1)$$
$$= -2 - 2 + 15 = 11$$

$$B = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 2 \\ 2 & 4 & 2 \end{bmatrix}$$

$$\Rightarrow \quad |B| = 2(4-8) - 3(6-4) + 4(12-4)$$
$$= -8 - 6 + 32 = 18$$

$$\therefore \quad \sqrt{|\text{adj } AB|} = \sqrt{|\text{adj } A||\text{ adj } B|}$$
$$= \sqrt{|A|^{3-1}|B|^{3-1}}$$
$$= |A||B| = 11(18) = 198$$

Question7

If $A = \begin{bmatrix} 1 & 5 & 2 \\ 4 & 1 & 3 \\ 2 & 6 & 3 \end{bmatrix}$, then $|(\text{adj } A)^{-1}| =$

Options:

A. -1

B. 1

C. 4

D. -4

Answer: B

Solution:

Step 1: Find the determinant of A

Matrix A is: $A = \begin{bmatrix} 1 & 5 & 2 \\ 4 & 1 & 3 \\ 2 & 6 & 3 \end{bmatrix}$

So, $|A| = 1(3-18) - 5(12-6) + 2(24-2)$

Calculate each part:

- $1 \times (3-18) = 1 \times (-15) = -15$
- $-5 \times (12-6) = -5 \times 6 = -30$
- $2 \times (24-2) = 2 \times 22 = 44$

Add them up: $|A| = -15 - 30 + 44 = -1$

Step 2: Find the value of $\left|(\text{adj } A)^{-1}\right|$

$\left|(\text{adj } A)^{-1}\right| = \frac{1}{|\text{adj } A|}$

The order of matrix A is 3, so: $|\text{adj } A| = |A|^{3-1} = |A|^2$

So, $\left|(\text{adj } A)^{-1}\right| = \frac{1}{|A|^2}$

Since $|A| = -1$, $\left|(\text{adj } A)^{-1}\right| = \frac{1}{(-1)^2} = \frac{1}{1} = 1$

Question8

The amplitude of the complex number $\frac{(\sqrt{3}+i)(1-\sqrt{3}i)}{(-1+i)(-1-i)}$ is

Options:

A.

$\frac{\pi}{2}$

B.

$\frac{\pi}{3}$

C.

$-\frac{5\pi}{12}$

D.

$-\frac{\pi}{6}$

Answer: D

Solution:

We have,

$$= \frac{(\sqrt{3}+i)(1-\sqrt{3}i)}{(-1+i)(-1-i)}$$
$$= \frac{\sqrt{3}-3i+i+\sqrt{3}}{1+i-i+1} = \frac{2\sqrt{3}-2i}{2}$$
$$= \sqrt{3}-i$$

$$\operatorname{Amp}(z) = -\tan^{-1}\left|\frac{1}{\sqrt{3}}\right| = \frac{-\pi}{6}$$

Question9

If a complex number $z = x + iy$ represents a point $p(x, y)$ in the argand plane and z satisfies the condition that the imaginary part of $\frac{z-3}{z+3i}$ is zero, then the locus of the point P is

Options:

A.

$x^2 + y^2 - 3x + 3y = 0, (x, y) \neq (0, -3)$

B.

$2xy - 3x + 3y + 9 = 0, (x, y) \neq (0, -3)$

C.

$x - y - 3 = 0, (x, y) \neq (0, -3)$

D.

$x + y + 3 = 0, (x, y) \neq (0, -3)$

Answer: C

Solution:

We have,

$$\operatorname{Im}\left(\frac{z-3}{z+3i}\right)=0, z+3i\neq 0$$

$$\therefore x\neq 0, y\neq -3$$

$$\Rightarrow \quad \frac{z-3}{z+3i}-\left(\overline{\frac{z-3}{z+3i}}\right)=0$$

$$\Rightarrow \quad \frac{z-3}{z+3i}-\frac{\bar{z}-3}{\bar{z}-3i}=0$$

$$\Rightarrow \quad (z-3)(\bar{z}-3i)-(z+3i)(\bar{z}-3)=0$$

$$\Rightarrow \quad z\bar{z}-3iz-3\bar{z}+9i-z\bar{z}+3z-3i\bar{z}+9i=0$$

$$\Rightarrow \quad -3i(z+\bar{z})+3(z-\bar{z})+18i=0$$

$$\because \quad z=x+iy, \bar{z}=x-iy$$

$$\Rightarrow \quad -3i(2x)+3(2iy)+18i=0$$

$$\Rightarrow \quad x-y-3=0, x, y\neq(0,-3)$$

Question10

$$(\sqrt{3}+i)^{10}+(\sqrt{3}-i)^{10}=$$

Options:

A.

$1024\sqrt{3}$

B.

1024

C.

2048

D.

$512\sqrt{3}$

Answer: B

Solution:

We have,

$$z=(\sqrt{3}+i)^{10}+(\sqrt{3}-i)^{10}$$
$$=\left(2e^{\frac{\pi}{6}}\right)^{10}+\left(2e^{\frac{-\pi i}{6}}\right)^{10}$$
$$=2^{10}\left(e^{\frac{10\pi i}{6}}+e^{\frac{-i10\pi}{6}}\right)$$
$$=2^{10}\left(\cos\frac{10\pi}{6}+i\sin\frac{10\pi}{6}+\cos\frac{10\pi}{6}-i\sin\frac{10\pi}{6}\right)$$
$$=2^{10}\cdot 2\cos\frac{5\pi}{3}=2^{11}\cos\left(2\pi-\frac{\pi}{3}\right)$$
$$=2^{11}\times\frac{1}{2}=2^{10}=1024$$

Question11

Number of real values of $(-1-\sqrt{3i})^{3/4}$ is

Options:

A.

0

B.

1

C.

2

D.

3

Answer: C

Solution:

We have,

$$Let\ (-1-\sqrt{3}i)^{3/4}$$
$$z=-1-\sqrt{3}i$$
$$z=2e^{i\left(\frac{-2\pi}{3}\right)}$$
$$z^{3/4}=e^{i\left(\frac{-2\pi}{3}\right)}$$
$$=\left(2e^{-i\frac{2\pi}{3}}\right)^{3/4}$$
$$=2^{3/4}\left(e^{-i\pi}\right)^{1/2}$$
$$\therefore \text{At}\quad k=0, z_1=2^{3/4}e^{0}=2^{3/4}$$
$$\text{At}\quad k=1, z_2=2^{3/4}e^{-i\pi}=-2^{3/4}$$

Number of real value $=2$

Question12

If $\tan\theta$ and $\cot\theta$ are two distinct roots of the equation $ax^2 + bx + c = 0, a \neq 0, b \neq 0$, then

Options:

A.

$\cos 2\theta = -\frac{2b}{c}$

B.

$\sin 2\theta = -\frac{2c}{b}$

C.

$\tan 2\theta = \frac{2b}{c}$

D.

$\cot 2\theta = \frac{2c}{a}$

Answer: B

Solution:

The equation $ax^2 + bx + c = 0$ has the roots $\tan\theta$ and $\cot\theta$.

For a quadratic equation $ax^2 + bx + c = 0$, if p and q are roots, then:

Sum of roots: $p + q = -\frac{b}{a}$
Product of roots: $pq = \frac{c}{a}$

Here, the roots are $\tan\theta$ and $\cot\theta$. So, $\tan\theta + \cot\theta = -\frac{b}{a}$ and $\tan\theta \cdot \cot\theta = \frac{c}{a}$.

$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

This simplifies to: $\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$,
because $\sin^2\theta + \cos^2\theta = 1$

Now, $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$
But from earlier, $\tan\theta + \cot\theta = -\frac{b}{a}$.
So, $\frac{1}{\sin\theta\cos\theta} = -\frac{b}{a}$.

For $\tan\theta \cdot \cot\theta = \frac{c}{a}$:
$\tan\theta \cdot \cot\theta = 1$, so $\frac{c}{a} = 1 \implies c = a$.

Now, take the result $\frac{1}{\sin\theta\cos\theta} = -\frac{b}{a}$.
Flip both sides: $\sin\theta\cos\theta = -\frac{a}{b}$

$2\sin\theta\cos\theta = 2\times\left(-\frac{a}{b}\right) = -\frac{2a}{b}$

But $2\sin\theta\cos\theta = \sin 2\theta$.
So, $\sin 2\theta = -\frac{2a}{b}$.

Since $a = c$, we can also write $\sin 2\theta = -\frac{2c}{b}$.

Question13

Sum of all the roots of the equation $||2x-3|-4| = 2$ is

Options:

A.

8

B.

0

C.

6

D.

9

Answer: C

Solution:

We have,

$$\begin{aligned} & ||2x-3|-4| = 2 \\ \Rightarrow\ & |2x-3|-4 = \pm 2 \\ \Rightarrow\ & |2x-3| = \pm 2+4 \\ \Rightarrow\ & |2x-3| = 6 \text{ or } |2x-3| = 2 \\ \Rightarrow\ & 2x-3 = \pm 6 \text{ or } 2x-3 = \pm 2 \\ \Rightarrow\ & 2x = \pm 6+3 \text{ or } 2x = \pm 2+3 \\ \Rightarrow\ & 2x = 9, -3 \text{ or } 2x = 5, 1 \\ \Rightarrow\ & x = \tfrac{9}{2}, \tfrac{-3}{2} \text{ or } x = \tfrac{5}{2}, \tfrac{1}{2} \end{aligned}$$

$\therefore$ Sum of roots

$$= \frac{9}{2} - \frac{3}{2} + \frac{5}{2} + \frac{1}{2}$$
$$= \frac{9-3+5+1}{2} = \frac{12}{2} = 6$$

Question14

If the quotient and remainder obtained when the expression $3x^5 - 6x^4 + 2x^3 + 4x^2 - 5x + 8$ is divided by the expression $x^2 - 2x + 3$ are $ax^3 + bx^2 + cx + d$ and $px + q$ respectively, then $ab + cd =$

Options:

A.

$p + 2q$

B.

$p + 2q - 2$

C.

$2p + q$

D.

$2p + q - 2$

Answer: B

Solution:

We have,

$$\begin{aligned}
& 3x^5 - 6x^4 + 2x^3 + 4x^2 - 5x + 8 \\
\Rightarrow\ & 3x^3\left(x^2 - 2x + 3\right) - 7x\left(x^2 - 2x + 3\right) - 10\left(x^2 - 2x + 3\right) - 4x + 38 \\
\Rightarrow\ & \left(x^2 - 2x + 3\right)\left(3x^3 - 7x - 10\right) - 4x + 38 \\
\therefore\ & a = 3, b = 0, c = -7, d = -10 \\
& p = -4, q = 38 \\
\therefore\ & ab + cd = 70 \\
& p + 2q - 2 = -4 + 76 - 2 = 70 \\
\therefore\ & ab + cd = p + 2q - 2
\end{aligned}$$

Question15

If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $12x^4 - 56x^3 + 89x^2 - 56x + 12 = 0$ such that $\alpha\beta = \gamma\delta = 1$ and $\frac{\alpha+\beta}{\gamma+\delta} > 1$, then $\frac{\alpha+\beta}{\gamma+\delta} =$

Options:

A.

$\frac{65}{6}$

B.

$\frac{13}{2}$

C.

$\frac{17}{15}$

D.

$\frac{15}{13}$

Answer: D

Solution:

$$12x^4 - 56x^3 + 89x^2 - 56x + 12 = 0 \quad \text{roots: } \alpha, \beta, \gamma, \delta$$

$\alpha\beta = \alpha\delta = 1$

$12\left(x^2 + \frac{1}{x^2}\right) - 56\left(x + \frac{1}{x}\right) + 89 = 0$

$12\left(x + \frac{1}{x}\right)^2 - 24 - 56\left(x + \frac{1}{x}\right) + 89 = 0$

$12\left(x + \frac{1}{x}\right)^2 - 56\left(x + \frac{1}{x}\right) + 65 = 0$

$12t^2 - 56t + 65 = 0, \quad x + \frac{1}{x} = t$

$t = \frac{56 \pm \sqrt{(56)^2 - 4(12)(65)}}{2 \times 12}$

$$= \frac{54 \pm 4}{24} \quad \begin{cases} + : \frac{60}{24} = \frac{5}{2} \\ - : \frac{52}{24} = \frac{13}{6} \end{cases}$$

$$x+\frac{1}{x}=\frac{5}{2}, x+\frac{1}{x}=\frac{13}{6}$$
$$x=2, \frac{1}{2}, x=\frac{2}{3}, \frac{3}{2}$$
$$\therefore \frac{\alpha+\beta}{\gamma+\delta}=\frac{\frac{5}{2}}{\frac{13}{6}}=\frac{15}{13}$$

Question16

If all the letters of the word ACADEMICIAN are permuted in all possible ways, then the number of permutations in which no two $A's$ are together and all the consonants are together is

Options:

A.

7200

B.

14400

C.

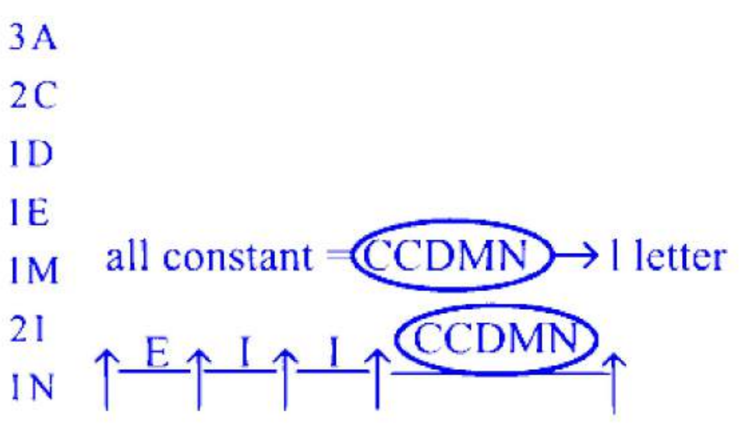
3600

D.

1800

Answer: A

Solution:



Arrangement $=\frac{4!}{2!}$

Selection of 3 gaps out of 5 for $3A'5={}^5C_3$

∴ Required arrangements

$$= \frac{4!}{2!} \cdot {}^5C_3 \cdot 1 \times \frac{5!}{2!}$$
$$= \frac{4!}{2!} \times \frac{5 \times 4}{2 \times 1} \times \frac{5!}{2!}$$
$$= 12 \times 5 \times 120 = 7200$$

Question17

The number of all possible three letter words that can be formed by choosing three letters from the letters of the word FEBRUARY so that a vowel always occupies the middle place is

Options:

A.

90

B.

93

C.

126

D.

129

Answer: B

Solution:

Selecting 1 vowel 2 consonants (arrangement in which vowel is at middle place)

$$= \left({}^3C_1 \cdot {}^4C_2\right)2! + \left({}^3C_1 \cdot {}^1C_1\right) \cdot 1$$
$$= 3 \times 6 \times 2 + 3 = 39$$

Similarly,

Selecting, 2 vowels 1 consonant

$= {}^3C_2 \cdot {}^4C_1(2!) \times 2 = 3 \times 4 \times 4 = 48$

Selecting 3 vowels $= 3! = 6$

Total words $= 39 + 48 + 6 = 93$

Question18

The number of ways in which 6 boys and 4 girls can be arranged in a row such that between any two girls there must be exactly 2 boys is

Options:

A.

6!5!

B.

(72)6!

C.

(144)5 !

D.

4!7!

Answer: C

Solution:

6 Boys and 4 girls

Number of arrangements in a row such that between any 2 girls there must be exactly 2 boys is

$$= \xrightarrow{G_1} \uparrow \xrightarrow{G_2} \uparrow \xrightarrow{G_3} \uparrow \xrightarrow{G_4}$$
$$= 4!\left(\frac{6!}{(2!)^3 \cdot 3!}\right) \cdot 3!(2!)^3$$
$$= 6! \times 24 = 144(5!)$$

Question19

If $C_0, C_1, C_2, \ldots, C_n$ are the binomial coefficients in the expansion of $(1+x)^n$ then the value of $\Sigma r^3 \cdot C_r$ when $n = 5$ is

Options:

A.

320

B.

560

C.

720

D.

800

Answer: D

Solution:

We have,

$$= \sum_{r=0}^{5} r^3 \cdot {}^5C_r = \sum_{r=1}^{5} r^3 \cdot \frac{5}{r} \cdot {}^4C_{r-1}$$
$$= \sum_{r=1}^{5} r^2 \cdot 5 \cdot {}^4C_{r-1} = t = r - 1$$
$$= 5 \sum_{t=0}^{4} (t+1)^2 \cdot {}^4C_t$$
$$5\left(\Sigma t^2 \cdot {}^4C_t + 2\Sigma t \cdot {}^4C_t + \Sigma^4 C_t\right)$$
$$= 5\left(\Sigma t^2 \cdot \frac{4}{t} \cdot {}^3C_{t-1} + 2\Sigma t \cdot \frac{4}{t} \cdot {}^3C_{t-1} + 2^t\right)$$
$$= 5\left(4\Sigma t^3 C_{t-1} + 8\left(2^3\right) + 2^4\right)$$
$$p = t - 1$$
$$= 5\left(4 \cdot \Sigma(p+1)^3 C_p + 64 + 16\right)$$
$$= 5\left(4 \cdot \Sigma p \cdot {}^3C_p + 4\left(2^3\right) + 64 + 16\right)$$
$$= 5\left(4 \cdot 3 \cdot 2^2 + 32 + 64 + 16\right)$$
$$= 5(48 + 32 + 80) = 5(160) = 800$$

Question20

The coefficient of x^{12} in the expansion of $\left(x^2 + 2x + 2\right)^8$ is

Options:

A.

1120

B.

2240

C.

2576

D.

4152

Answer: C

Solution:

We need to find the coefficient of x^{12} in the expansion of $\left(x^2+2x+2\right)^8$.

Step 1: General Term Formula

Each term in the expansion is made by choosing P_1 times x^2, P_2 times $2x$, and P_3 times 2 from the 8 factors, so $P_1+P_2+P_3=8$. The general term is:
$$\frac{8!}{P_1!P_2!P_3!}(x^2)^{P_1}(2x)^{P_2}(2)^{P_3}$$

Step 2: Power of x

The power of x in each term is $2P_1$ (from x^2) plus P_2 (from $2x$), for a total of $2P_1+P_2$.

Step 3: Find Values for x^{12}

We want $2P_1+P_2=12$ and $P_1+P_2+P_3=8$.

Possible solutions are:

P_1	P_2	P_3
6	0	2
5	2	1
4	4	0

Step 4: Write Each Term's Coefficient

For each solution, we substitute into the formula:

- When $P_1=6,\ P_2=0,\ P_3=2$:
  Coefficient: $\frac{8!}{6!0!2!}\cdot 2^0\cdot 2^0\cdot 2^2$
- When $P_1=5,\ P_2=2,\ P_3=1$:
  Coefficient: $\frac{8!}{5!2!1!}\cdot 2^2\cdot 2^1$
- When $P_1=4,\ P_2=4,\ P_3=0$:
  Coefficient: $\frac{8!}{4!4!0!}\cdot 2^4$

Step 5: Calculate Coefficients

Total Coefficient
$$=\frac{8!}{6!2!}\cdot 2^2+\frac{8!}{5!2!1!}\cdot 2^2\cdot 2^1+\frac{8!}{4!4!}\cdot 2^4$$
$$=28\times 4+168\times 4\times 2+70\times 16$$
$$=112+1344+1120$$
$$=2576$$

The coefficient of x^{12} is $\boxed{2576}$.

Question21

If $\frac{x^2+1}{(x^2+2)(x^2+3)}=\frac{Ax+B}{x^2+2}+\frac{Cx+D}{x^2+3}$**, then** $A+B+C+D=$

Options:

A.

0

B.

1

C.

-1

D.

6

Answer: B

Solution:

$$\frac{x^2+1}{(x^2+2)(x^2+3)}=\frac{Ax+B}{x^2+2}+\frac{Cx+D}{x^2+3}$$
$$x^2+1=(Ax+B)\left(x^2+3\right)+(Cx+D)\left(x^2+2\right)$$

Coefficient of $x^3=0$, coefficient of $x=0$

$\Rightarrow A+C=0, 3A+2C=0 \Rightarrow A=C=0$

Coefficient of $x^2=1$, constant term $=1$

$B+D=1, \quad 1=3B+2D$

$$\begin{array}{l} 3B + 2D = 1 \\ 2B + 2D = 2 \\ \underline{- \quad - \quad -} \\ B = -1, \ D = 2 \end{array}$$

$\therefore A+B+C+D=1$

Question22

If $2\sin\theta+3\cos\theta=2$ **and** $\theta\neq(2n+1)\frac{\pi}{2}$**, then** $\sin\theta+\cos\theta=$

Options:

A.

$5/13$

B.

$3/5$

C.

$7/13$

D.

$4/5$

Answer: C

Solution:

$$
\begin{aligned}
& 2\sin\theta + 3\cos\theta = 2 \\
& \Rightarrow (2\sin\theta + 3\cos\theta)^2 = 4 \\
& \Rightarrow 4\sin^2\theta + 9\cos^2\theta + 12\sin\theta\cos\theta = 4 \\
& \Rightarrow 5\cos^2\theta + 12\sin\theta\cos\theta = 0 \\
& \Rightarrow \cos\theta(5\cos\theta + 12\sin\theta) = 0 \\
& \Rightarrow \tan\theta = \frac{-5}{12} \\
& \sin\theta = \frac{-5}{13}, \cos\theta = \frac{12}{13} \\
& \therefore \sin\theta + \cos\theta = \frac{-5}{13} + \frac{12}{13} = \frac{7}{13}
\end{aligned}
$$

Question23

If $\sin A = -\frac{24}{25}$, $\cos B = \frac{15}{17}$, A does not belong to 4th quadrant and B does not belong to 1st quadrant, then $(A+B)$ lies in the quadrant

Options:

A.

1st quadrant

B.

2 nd quadrant

C.

3rd quadrant

D.

4th quadrant

Answer: C

Solution:

$\sin A = \frac{-24}{25}, \cos B = \frac{15}{17}$

According to the question,

A is 3rd quadrant, B in 4th quadrant.

Now, $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$= \frac{-24}{25} \times \frac{15}{17} + \left(\frac{-7}{25}\right) \cdot \left(\frac{-8}{17}\right)$$
$$\Rightarrow \quad \frac{8(-45+7)}{25(17)} \Rightarrow \frac{-37 \times 8}{25 \times 17}$$
$$\therefore \sin(A+B) < 0; \pi < A < \frac{3\pi}{2}, \frac{3\pi}{2} < B < 2\pi$$
$$\frac{5\pi}{2} < A + B < \frac{7\pi}{2}$$

$\therefore A + B$ will be in 3rd quadrant.

Question24

$4 \cos \frac{7\theta}{2} \cos \frac{3\theta}{2} \sin 5\theta =$

Options:

A.

$\sin 10\theta + \sin 7\theta - \sin 3\theta$

B.

$\sin 10\theta + \sin 7\theta - \sin 5\theta$

C.

$\sin 10\theta + \sin 7\theta + \sin 3\theta$

D.

$\sin 10\theta + \sin 7\theta + \sin 5\theta$

Answer: C

Solution:

We have,

$$4\cos\frac{7\theta}{2}\cos\frac{3\theta}{2}\sin 5\theta$$
$$= 2(\cos 5\theta + \cos 2\theta)\sin 5\theta$$
$$(\because 2\cos A\cos B = \cos(A+B) + \cos(A-B))$$
$$= 2\sin 5\theta\cos 5\theta + 2\sin 5\theta\cos 2\theta$$
$$= \sin 10\theta + \sin 7\theta + \sin 3\theta$$
$$(\because 2\sin A\cos B = \sin(A+B) + \sin(A-B))$$

Question25

If $x \in (-\pi, \pi)$, then the number of solutions of the equation $2\sin x\sin 3x\sin 5x + \sin 5x\cos 4x = 0$ is

Options:

A.

14

B.

12

C.

13

D.

9

Answer: C

Solution:

$$2\sin x\sin 3x\sin 5x + \cos 4x\sin 5x = 0$$
$$= \sin 5x(2\sin x\sin 3x + \cos 4x) = 0$$
$$= \sin 5x(2\sin x\sin 3x + \cos x\cos 3x - \sin x\sin 3x) = 0$$
$$= \sin 5x(\cos x\cos 3x + \sin x\cdot\sin 3x) = 0$$
$$= \sin 5x\cdot\cos 2x = 0$$

$\therefore = \sin 5x = 0 \qquad \cos 2x = 0$

$= 5x = n\pi \qquad 2x = (2k+1)\frac{\pi}{2}$

$x = \frac{n\pi}{5} \qquad x = (2k+1)\frac{\pi}{4}$

$n \in I \qquad k \in I$

$\because x \in (-\pi, \pi)$

$-\pi < \frac{n\pi}{5} < \pi \qquad -\pi < (2k+1)\frac{\pi}{4} < \pi$

$-5 < n < 5 \qquad -4 < 2k+1 < 4$

$\Rightarrow \frac{-5}{2} < k < \frac{3}{2}$

Number of values of x is

$9 + 4 = 13$

Question26

The number of values of x satisfying the equation, $\tan^{-1}\left(x + \frac{\sqrt{2}}{x}\right) + \tan^{-1}\left(x - \frac{\sqrt{2}}{x}\right) = \tan^{-1}(x)$ is

Options:

A.

0

B.

1

C.

2

D.

3

Answer: C

Solution:

We have,

$$\tan^{-1}\left(x+\frac{\sqrt{2}}{x}\right)+\tan^{-1}\left(x-\frac{\sqrt{2}}{x}\right)$$
$$=\tan^{-1}x$$
$$x\neq 0$$
$$\text{Now, }\tan^{-1}\left(\frac{x+\frac{\sqrt{2}}{x}+x-\frac{\sqrt{2}}{x}}{1-\left(x+\frac{\sqrt{2}}{x}\right)\left(x-\frac{\sqrt{2}}{x}\right)}\right)$$
$$=\tan^{-1}x$$
$$\Rightarrow \quad \frac{2x}{1-\left(x^2-\frac{2}{x^2}\right)}=x$$
$$\Rightarrow \quad 2=1-x^2+\frac{2}{x^2}$$
$$x^2-\frac{2}{x^2}+1=0$$
$$\Rightarrow \quad x^4+x^2-2=0$$
$$\left(x^2+2\right)\left(x^2-1\right)=0$$
$$x^2=2\Rightarrow x=\pm 1$$

$\therefore$ Number of values of $x=2$

Question27

$\cot h^2x-\tanh^2x=$

Options:

A.

$4\operatorname{cosech}2x\tanh 2x$

B.

$4\operatorname{sech}2x\coth 2x$

C.

$4\operatorname{sech}2x\tanh 2x$

D.

$4\cosh 2x(\operatorname{cosech}2x)^2$

Answer: D

Solution:

$\coth^2 x-\tanh^2 x$
$=\dfrac{\cosh^2 x}{\sinh^2 x}-\dfrac{\sinh^2 x}{\cosh^2 x}$
$=\dfrac{\left(\cosh^2 x+\sinh^2 x\right)\left(\cosh^2 x-\sinh^2 x\right)}{\sinh^2 x\cdot\cosh^2 x}$
$=\dfrac{\left(\cosh^2 x+\sinh^2 x\right)}{\sinh^2 x\cdot\cosh^2 x}=\dfrac{4\cosh 2x}{(\sinh 2x)^2}$
$=4\cosh 2x(\operatorname{cosech} 2x)^2$

Question28

If $a=3, b=5, c=7$ are the sides of a $\triangle ABC$, then its circumradius is

Options:

A.

$\frac{7}{\sqrt{3}}$

B.

$\frac{15}{2}$

C.

$\frac{15\sqrt{3}}{4}$

D.

$\frac{\sqrt{3}}{2}$

Answer: A

Solution:

$a=3, b=5, c=7, s=\dfrac{15}{2}$

$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{\dfrac{15}{2}\left(\dfrac{15}{2}-3\right)\left(\dfrac{15}{2}-5\right)\left(\dfrac{15}{2}-7\right)}$

$$= \frac{1}{4}\sqrt{15 \times 9 \times 5 \times 1} = \frac{5 \times 3}{4}\sqrt{3}$$

$$\Rightarrow \quad \frac{15}{4}\sqrt{3}$$

$$\text{Circum radius}(R) = \frac{abc}{4\Delta}$$

$$= \frac{3 \times 5 \times 7}{4\left(\frac{15}{4}\sqrt{3}\right)} = \frac{7}{\sqrt{3}}$$

Question29

Two ships leave a port at the same time. One of them move in the direction of $E 50^{\circ} \mathrm{N}$ with a speed of 8 kmph and the other moves in the direction of $\mathrm{S} 20^{\circ} \mathrm{E}$ with a speed of 12 kmph . Then, the distance between the ships at the end of 2 h is (in km)

Options:

A.

$8\sqrt{7}$

B.

34

C.

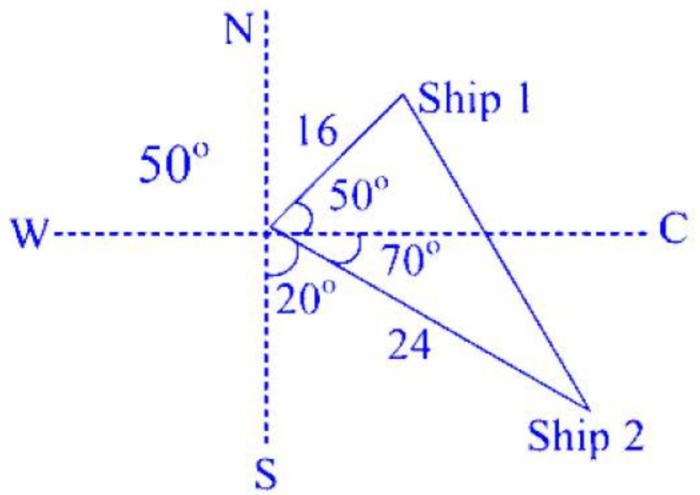
$8\sqrt{19}$

D.

32

Answer: C

Solution:



Step 1: Find how far each ship goes in 2 hours.

The first ship travels at 8 km per hour. In 2 hours, it travels:
$8 \times 2 = 16$ km

The second ship travels at 12 km per hour. In 2 hours, it travels:
$12 \times 2 = 24$ km

Step 2: Find the angle between the directions of the ships.

The paths of the two ships make a 120° angle with each other. We use this angle to find the distance between them after 2 hours.

Step 3: Use the Cosine Law to find the distance between ships.

The Cosine Law formula is: $c^2 = a^2 + b^2 - 2ab\cos C$ where a and b are the distances traveled, and C is the angle between the paths.

Let $a = 24$ km, $b = 16$ km, and $C = 120^\circ$.

Plug these values into the formula:

$\cos 120^\circ = \frac{24^2+16^2-c^2}{2\times24\times16}$

Since $\cos 120^\circ = -\frac{1}{2}, -\frac{1}{2} \times 2 \times 24 \times 16 = 24^2 + 16^2 - c^2$

$24^2 = 576, 16^2 = 256$, and $2 \times 24 \times 16 = 768$. So, $-\frac{1}{2} \times 768 = 576 + 256 - c^2 - 384 = 832 - c^2$

Add c^2 on both sides and add 384 to both sides: $c^2 = 832 + 384 = 1216$

Now, $1216 = 64 \times 19$. So, $c = 8\sqrt{19}$

So, the distance between the ships after 2 hours is $8\sqrt{19}$ km.

Question30

In a $\triangle ABC$, if $\mathbf{BC} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{CA} = 6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, then the perimeter of the triangle is

Options:

A.

$5(2+\sqrt{3})$

B.

$5(2+\sqrt{2})$

C.

$\sqrt{10}(3+\sqrt{10})$

D.

$10(2+\sqrt{5})$

Answer: B

Solution:

In $\triangle ABC$,

$$\begin{aligned}
&\mathbf{BC} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}} \Rightarrow BC = \sqrt{9} = 3 \\
&\mathbf{CA} = 6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}} \\
\Rightarrow \quad & CA = \sqrt{36+9+4} = 7 \\
\Rightarrow \quad & \mathbf{AC} = -6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}} \\
\Rightarrow \quad & \mathbf{AB} = \mathbf{AC} - \mathbf{BC} = -7\hat{\mathbf{i}} - \hat{\mathbf{j}} \\
& AB = \sqrt{49+1} = \sqrt{50} \\
\text{Perimeter } & = 3 + 7 + \sqrt{50} = 10 + \sqrt{50} \\
& = 10 + 5\sqrt{2} = 5(2+\sqrt{2})
\end{aligned}$$

Question31

$\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}, a_1\hat{\mathbf{i}}+b_1\hat{\mathbf{j}}+c_1\hat{\mathbf{k}}, a_2\hat{\mathbf{i}}+b_2\hat{\mathbf{j}}+c_2\hat{\mathbf{k}}, a_3\hat{\mathbf{i}}+b_3\hat{\mathbf{j}}+c_3\hat{\mathbf{k}}$ are the position vectors of the points A, B, C, D respectively. $\frac{2}{3}(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$ is the position vector of the centroid of the triangular face BCD of the tetrahedron $ABCD$. If $\alpha\hat{\mathbf{i}}+\beta\hat{\mathbf{j}}+\gamma\hat{\mathbf{k}}$ is the position vector of the centroid of the tetrahedron, then $2\alpha+\beta+\gamma=$

Options:

A.

3

B.

2

C.

$\frac{2}{3}$

D.

$\frac{3}{4}$

Answer: A

Solution:

Let position of A, B, C and D be $\mathbf{a}$, $\mathbf{b}$, c and d respectively.

$\mathbf{a}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$

In $\triangle BCD$ centroid is $\frac{\mathbf{b}+\mathbf{c}+\mathbf{d}}{3}$

$$\therefore \quad \frac{\mathbf{b}+\mathbf{c}+\mathbf{d}}{3}=\frac{2}{3}(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$$

$\mathbf{b}+\mathbf{c}+\mathbf{d}=2(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$

$\therefore$ Centroid of tetrahedron $ABCD$ is

$$=\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}}{4}=\frac{3(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})}{4}$$

$$\therefore \quad \alpha=\frac{3}{4}, \beta=\frac{3}{4}, \gamma=\frac{3}{4}$$

$$\therefore \quad 2\alpha+\beta+\gamma=\frac{6+3+3}{4}=\frac{12}{4}=3$$

Question32

If $\mathbf{a}=\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\mathbf{b}=9\hat{\mathbf{i}}+6\hat{\mathbf{j}}-18\hat{\mathbf{k}}$ are two vectors, then $\frac{\text{Projection of } \mathbf{b} \text{ on } \mathbf{a}}{\text{Projection of } \mathbf{a} \text{ on } \mathbf{b}}=$

Options:

A.

21

B.

7

C.

$\frac{7}{3}$

D.

3

Answer: B

Solution:

$$\begin{aligned} &\mathbf{a}=\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}} \text{ and} \\ &\quad \mathbf{b}=9\hat{\mathbf{i}}+6\hat{\mathbf{j}}-18\hat{\mathbf{k}}=3(3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-6\hat{\mathbf{k}}) \\ &\quad \therefore \frac{\mathbf{b}\cdot\mathbf{a}}{\text{Projection of } \mathbf{b} \text{ on } \mathbf{a}}=\frac{\frac{|\mathbf{a}|}{\mathbf{a}\cdot\mathbf{b}}}{|\mathbf{b}|}=\frac{|b|}{|a|} \\ &\quad =\frac{3\sqrt{9+4+36}}{\sqrt{1+4+4}}=7 \end{aligned}$$

Question33

Let $\mathbf{a}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}, \mathbf{b}=2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\mathbf{c}=3\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ be three vectors. If $\mathbf{r}$ is a vector such that $\mathbf{r}\cdot\mathbf{a}=0, \mathbf{r}\cdot\mathbf{b}=-2$ and $\mathbf{r}\cdot\mathbf{c}=6$, then $\mathbf{r}\cdot(\beta\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})=$

Options:

A.

0

B.

1

C.

2

D.

3

Answer: D

Solution:

Let $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

$\therefore \quad \mathbf{r} \cdot \mathbf{a} = 0$

$x + 2y + 3z = 0 \qquad \ldots(i)$

$\Rightarrow x = -2y - 3z$

And $\mathbf{r} \cdot \mathbf{b} = -2 \qquad \ldots(ii)$

$2x - 3y + z = -2$

$-4y - 6z - 3y + z = -2$

$-5z = -2 + 7y$

$$z = \frac{1}{5}(2 - 7y)$$

$$x = -2y - \frac{3}{5}(2 - 7y) = \frac{11y - 6}{5}$$

$and \quad \mathbf{r} \cdot \mathbf{C} = 6$

$3x + y - 2z = 6 \qquad \ldots(iii)$

From by Eq. (iii), we get

$\therefore \quad \frac{3}{5}(11y - 6) + y - \frac{2}{5}(2 - 7y) = 6$

$\Rightarrow \quad 33y - 18 + 5y - 4 + 14y = 30$

$\Rightarrow \quad 52y = 52$

$y = 1$

$x = 1$

$\therefore \quad r \cdot (3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

$= 3x + y + z = 3$

Question34

Let $\mathbf{a} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}, \mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}, \mathbf{c} = 6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ be three vectors. If $\mathbf{d}$ is a vector perpendicular to both $\mathbf{a}, \mathbf{b}$ and $|\mathbf{d} \times \mathbf{c}| = 14$, then $|\mathbf{d} \cdot \mathbf{c}| =$

Options:

A.

35

B.

70

C.

140

D.

105

Answer: B

Solution:

Given $\mathbf{a} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$
$\mathbf{c} = 6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

Vector $\mathbf{d}$ is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$

$$\therefore \quad \mathbf{d} = \lambda(\mathbf{a} \times \mathbf{b})$$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 1 \\ 1 & -2 & -2 \end{vmatrix} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

Let angle between $(\mathbf{a} \times \mathbf{b})$ and $\mathbf{c}$ be θ.

$$\begin{aligned} \therefore \cos\theta &= \frac{(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}}{|\mathbf{a} \times \mathbf{b}||\mathbf{c}|} \\ &= \frac{(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})}{\sqrt{26} \times 7} \\ &= \frac{24 + 9 + 2}{\sqrt{26} \times 7} = \frac{5}{\sqrt{26}} \end{aligned}$$

$$\therefore \sin\theta = \frac{1}{\sqrt{26}}$$

Now, $|\mathbf{d} \times \mathbf{c}| = 14$
$|\mathbf{d}||\mathbf{c}| \sin\theta = 14$
$\lambda|\mathbf{a} \times \mathbf{b}||\mathbf{c}| \sin\theta = 14 \quad [\because \hat{\mathbf{d}} = \lambda(\mathbf{a} \times \mathbf{b})]$
$$\lambda \times \sqrt{26} \times 7 \times \frac{1}{\sqrt{26}} = 14$$

$\lambda = 2$
$$\begin{aligned} \therefore |\mathbf{d} \cdot \mathbf{c}| &= |\lambda(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}| \\ &= 2(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) \\ &= 2(24 + 9 + 2) = 70 \end{aligned}$$

Question35

The mean deviation from the mean of the discrete data $2, 3, 5, 7, 11, 13, 17, 19, 22$ is

Options:

A.

8

B.

7.5

C.

5.5

D.

6

Answer: D

Solution:

$$\bar{X} = \frac{2+3+5+7+11+13+17+19+22}{9}$$
$$= 11$$
$|x_i - \bar{x}| = 9, 8, 6, 4, 0, 2, 6, 8, 11$
$\therefore$ Mean deviation from mean
$$= \frac{\Sigma |x_i - \bar{x}|}{9}$$
$$= \frac{9+8+6+4+0+2+6+8+11}{9}$$
$= \frac{54}{9} = 6$

Question36

Out of the given 25 consecutive position integers, three integers are drawn. If the least integer among given 25 integers is an odd number, then the probability that the sum of the three integers drawn is an even number is

Options:

A.

$\frac{289}{575}$

B.

$\frac{286}{575}$

C.

$\frac{288}{575}$

D.

$\frac{287}{575}$

Answer: A

Solution:

We have 25 numbers in a row, one after another, without skipping any numbers. The smallest number in this set is an odd number.

This means there are 13 odd numbers and 12 even numbers among them. (Odd and even numbers always alternate. If we start with an odd number, we will always have one more odd number than even numbers.)

We need to find the chance (probability) that if we pick 3 of these numbers, their total (sum) is an even number.

For the sum of 3 numbers to be even, two situations can happen:
1. We choose three even numbers. (Even + Even + Even = Even)
2. We choose two odd numbers and one even number. (Odd + Odd + Even = Even)

Counting the Ways

Number of ways to pick 3 even numbers:
There are 12 even numbers. Number of ways = ${}^{12}C_3$.

Number of ways to pick 2 odd numbers and 1 even number:
There are 13 odd numbers and 12 even numbers.
Ways to pick 2 odd numbers = ${}^{13}C_2$.
Ways to pick 1 even number = ${}^{12}C_1$.
Total ways = ${}^{13}C_2 \times^{12} C_1$.

Total Possible Ways

The total number of ways to select any 3 numbers from 25 numbers is ${}^{25}C_3$.

Probability Formula

The probability that the sum is even is:

$$P(A) = \frac{{}^{13}C_2 \cdot {}^{12}C_1 + {}^{12}C_3}{{}^{25}C_3} = \frac{289}{575}$$

Question37

If three dice are thrown at a time, then the probability of getting the sum of the numbers on them as a prime number is

Options:

A.

$\frac{3}{8}$

B.

$\frac{73}{216}$

C.

$\frac{4}{27}$

D.

$\frac{5}{54}$

Answer: B

Solution:

Let N be the total number of possible outcomes when three dice are thrown.

Each die has 6 faces, so the total number of outcomes is $N = 6 \times 6 \times 6 = 216$.

Let S be the sum of the numbers on the three dice.

The minimum possible sum is $1 + 1 + 1 = 3$.

The maximum possible sum is $6 + 6 + 6 = 18$.

We need to find the probability that the sum S is a prime number. The prime numbers between 3 and 18 (inclusive) are:

3, 5, 7, 11, 13, 17.

Now we need to count the number of ways to obtain each of these prime sums. Let (d_1, d_2, d_3) be the numbers on the three dice. The order matters (e.g., (1,1,2) is different from (1,2,1)).

1. **Sum = 3**:

   The only combination is (1, 1, 1).

   Number of ways: 1.

2. **Sum = 5**:

   Possible combinations (listing partitions in non-decreasing order to avoid duplicates, then counting permutations):

   - (1, 1, 3): Permutations: $\frac{3!}{2!1!} = 3$ ways ((1,1,3), (1,3,1), (3,1,1))
   - (1, 2, 2): Permutations: $\frac{3!}{1!2!} = 3$ ways ((1,2,2), (2,1,2), (2,2,1))

   Number of ways: $3 + 3 = 6$.

3. **Sum = 7**:

   Possible combinations ($d_1 \leq d_2 \leq d_3$):

   - (1, 1, 5): 3 ways
   - (1, 2, 4): 3! = 6 ways
   - (1, 3, 3): 3 ways
   - (2, 2, 3): 3 ways

   Number of ways: $3 + 6 + 3 + 3 = 15$.

4. **Sum = 11**:

   Possible combinations ($d_1 \leq d_2 \leq d_3$):

   - (1, 4, 6): 3! = 6 ways

- (1, 5, 5): 3 ways
- (2, 3, 6): 3! = 6 ways
- (2, 4, 5): 3! = 6 ways
- (3, 3, 5): 3 ways
- (3, 4, 4): 3 ways

Number of ways: $6 + 3 + 6 + 6 + 3 + 3 = 27$.

5. **Sum = 13**:

   Possible combinations ($d_1 \leq d_2 \leq d_3$):

   - (1, 6, 6): 3 ways
   - (2, 5, 6): 3! = 6 ways
   - (3, 4, 6): 3! = 6 ways
   - (3, 5, 5): 3 ways
   - (4, 4, 5): 3 ways

   Number of ways: $3 + 6 + 6 + 3 + 3 = 21$.

6. **Sum = 17**:

   Possible combinations ($d_1 \leq d_2 \leq d_3$):

   - (5, 6, 6): 3 ways

   (Note: sums like (4,x,y) would give max 4+6+6=16, so 5 must be present)

   Number of ways: 3.

Now, we sum the number of ways for each prime sum to get the total number of favorable outcomes:

Total favorable outcomes = $1 + 6 + 15 + 27 + 21 + 3 = 73$.

The probability of getting a sum that is a prime number is the ratio of favorable outcomes to the total possible outcomes:

Probability = $\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{73}{216}$.

Question38

Three companies C_1, C_2, C_3 produce car tyres. A car manufacturing company buys 40% of its requirement from $C_1, 35\%$ from C_2 and 25% from C_3. The company knows that 2% of the tyres supplied by $C_1, 3\%$ by C_2 and 4% by C_3 are defective. If a tyre chosen random from the consignment received is found defective then, the probability that it was supplied by C_2 is

Options:

A.

$\frac{7}{19}$

B.

$\frac{12}{19}$

C.

$\frac{10}{57}$

D.

$\frac{26}{57}$

Answer: A

Solution:

$E_1 =$ company buys tyres from c_1

$E_2 =$ company buys tyres from c_2

$E_3 =$ company buys tyres from c_3

$E =$ tyre is defective

According to the question,

$P\left(\frac{E_2}{E}\right) = ?$

Using Baye's theorem,

$$P\left(\frac{E_2}{E}\right) = \frac{P(E_2) \cdot P\left(\frac{E}{E_2}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) P\left(\frac{E}{E_2}\right)} + P(E_3) \cdot P\left(\frac{E}{E_3}\right)$$

$$= \frac{\frac{35}{100} \times \frac{3}{100}}{\frac{40}{100} \times \frac{2}{100} + \frac{35}{100} + \frac{3}{100} + \frac{25}{100} \times \frac{4}{100}}$$
$$= \frac{35 \times 3}{40 \times 2 + 35 \times 3 + 25 \times 4}$$
$$= \frac{21}{16 + 21 + 20} = \frac{21}{57} = \frac{7}{19}$$

Question39

The probability distribution of a random variable X is given below. Then, the standard deviation of X is

$X = x_1$	2	3	5	7	12
$P(X = x_1)$	$3k$	k	k	$2k$	k

Options:

A.

5

B.

11

C.

$\sqrt{11}$

D.

$\sqrt{5}$

Answer: C

Solution:

Step 1: List all values and probabilities

We have X taking the values 2, 3, 5, 7, and 12. Their probabilities are $3k$, k, k, $2k$, and k.

Step 2: Make a table with useful information

x_i	p_i	$p_i x_i$	x_i^2	$p_i x_i^2$
2	$3k$	$6k$	4	$12k$
3	k	$3k$	9	$9k$
5	k	$5k$	25	$25k$
7	$2k$	$14k$	49	$98k$
12	k	$12k$	144	$144k$
		$40k$		$288k$

Step 3: Find value of k

Add up all probabilities:

$3k + k + k + 2k + k = 8k$

Set equal to 1:

$8k = 1$

So, $k = \frac{1}{8}$

Step 4: Find the mean and expected value of x^2

$\Sigma p_i x_i = 40k$

$\Sigma p_i x_i^2 = 288k$

Step 5: Write the formula for standard deviation

$\text{SD} = \sqrt{\Sigma p_i x_i^2 - (\Sigma p_i x_i)^2}$

Step 6: Substitute the values and solve

$\text{SD} = \sqrt{288k - (40k)^2} = \sqrt{\frac{288}{8} - (5)^2} = \sqrt{36 - 25} = \sqrt{11}$

Question40

If the mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{10}{9}$ respectively, then $P(X \geq 6) =$

Options:

A.

$\frac{41}{6^8}$

B.

$\frac{741}{6^8}$

C.

$1 - \frac{741}{6^8}$

D.

$1 - \frac{41}{6^8}$

Answer: B

Solution:

$$\begin{aligned}
&\text{Mean } = np = \frac{4}{3} \\
&\text{Variance } = npq = \frac{10}{9} \\
&q = \frac{10}{9} \times \frac{3}{4} = \frac{5}{6} \\
&p = \frac{1}{6} \Rightarrow n = 8 \\
&\because p(x \geq 6) = p(x = 6) + p(x = 7) + p(x = 8) \\
&= {}^8C_6\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^2 + {}^8C_7\left(\frac{1}{6}\right)^7\left(\frac{5}{6}\right) + {}^8C_8\left(\frac{1}{6}\right)^8 \\
&= \frac{1}{6^8}\left({}^8C_6 \cdot 25 + 8(5) + 1\right) \\
&= \frac{1}{6^8}(28(29) + 41) \\
&= \frac{1}{6^8}(700 + 41) = \frac{741}{6^8}
\end{aligned}$$

Question41

A straight line passing through a point $(3,2)$ cuts X and Y axes at the points A and B respectively. If a point P divides AB in the ratio $2:3$, then the equation of the locus of point P is

Options:

A.

$\frac{9}{x}+\frac{4}{y}=1$

B.

$9x+4y=5xy$

C.

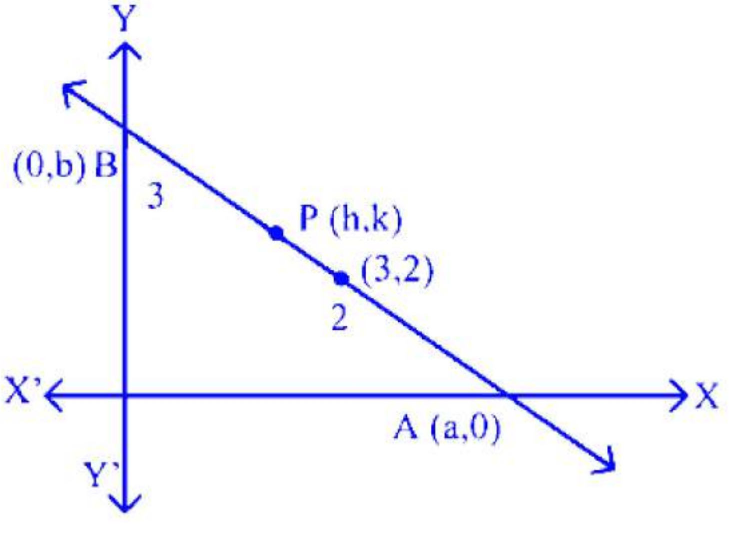
$4x+9y=5xy$

D.

$\frac{4}{x}+\frac{9}{y}=1$

Answer: C

Solution:



$h=\dfrac{3a}{5}, k=\dfrac{2b}{5}$

$\therefore$ Equation of AB

$\dfrac{3}{a}+\dfrac{2}{b}-1=0$

$\frac{3}{\frac{5h}{3}}+\frac{2}{5k}=1 \quad \left[\because h=\frac{3a}{5}, k=\frac{2b}{5}\right]$
$\frac{9}{h}+\frac{4}{k}=5$
Locus,
$\frac{9}{x}+\frac{4}{y}=5$
$4x+9y=5xy$

Question42

By shifting the origin to the point $(-1,2)$ through translation of axes, if $ax^2+2hxy+by^2+2gx+2fy+c=0$ is the transformed equation of $2x^2-xy+y^2-3x+4y-5=0$, then $2(f+g+h)=$

Options:

A.

$a+b+c$

B.

$a-5(b+c)$

C.

$3(a+b+c)$

D.

$c-5(a+b)$

Answer: D

Solution:

$X=x-h, Y=y-k$
$X=x+1, Y=y-2$
$x=X-1, y=Y+2$
$\Rightarrow \quad 2(X-1)^2-(X-1)(Y+2)+(Y+2)^2-3(X-1)+4(Y+2)-5=0$

$$\Rightarrow \quad 2X^2-4X+2-XY-2X+Y+2 \\ +Y^2+4y+4 \\ -3X+3+4Y+8-5=0$$

$2h=-1 \qquad a=2$
$2g=-4-2-3=-9 \qquad b=1$
$2f=1+4+4=9 \qquad c=2+2+4+3+8-5$
$=14$

$\therefore \quad 2(f+g+h)=-1$
And $\quad c-5(a+b)=14-15=-1$

Question43

If a line L passing through the point $A(-2,4)$ makes an angel of 60° with the positive direction of X - axis in anti-clockwise direction and $B(p,q)$ lying in the 3rd quadrant is a point on L at the distance of 6 units from the point A, then $\sqrt{p^2+q^2-8q}=$

Options:

A.

6

B.

7

C.

8

D.

9

Answer: A

Solution:

Using parametric form of line

$$\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=\pm r$$
$$\Rightarrow \frac{x+2}{\cos 60^\circ}=\frac{y-4}{\sin 60^\circ}=\pm 6$$
$$\Rightarrow x=\pm 6\cos 60^\circ-2, y=\pm 6\sin 60^\circ+4$$
$$x=\pm 6\left(\frac{1}{2}\right)-2y=\pm 3\sqrt{3}+4$$
$$=\pm 3-2$$

$\because$ Point lies in 3rd 24

$x=-5, y=-3\sqrt{3}+4$
$p=-5, q=4-3\sqrt{3}$

$\therefore \sqrt{p^2+q^2-8q}$
$= \sqrt{25+16+27-24\sqrt{3}-32+24\sqrt{3}}$
$= \sqrt{36} = 6$

Question44

If the perpendicular drawn from the point $(2, -3)$ to the straight line $4x - 3y + 8 = 0$ meets it at $M(a, b)$ and $a^3 - b^3 = k^3$, then $k =$

Options:

A.

1

B.

-1

C.

2

D.

-2

Answer: D

Solution:

Foot of perpendicular of point $(2, -3)$ with respect to line $4x - 3y + 8 = 0$

$$\frac{a-2}{4} = \frac{b+3}{-3} = \frac{-1(4(2)-3(-3)+8)}{4^2+3^2}$$
$$\Rightarrow \frac{a-2}{4} = \frac{b+3}{-3} = -1$$
$$\Rightarrow x = -2, b = 0$$
$$\therefore \quad a^3 - b^3 = k^3 \Rightarrow k = -2$$

Question45

Let Q be the image of a point $P(1, 2)$ with respect to the line $x + y + 1 = 0$ and R be the image of Q with respect to the line

$x-y-1=0$. If M and N are the mid-points of PQ and QR respectively, then $MN=$

Options:

A.

$\sqrt{10}$

B.

4

C.

$\sqrt{22}$

D.

5

Answer: A

Solution:

$Q(\alpha, \beta)$ and $R(\gamma, \delta), P(1,2)$

$$\therefore \frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{-2(1+2+1)}{1+1}$$

$$\Rightarrow \frac{\alpha-1}{1}=\frac{\beta-2}{1}=-4 \Rightarrow \alpha=-3, \beta=-2$$

$\therefore Q=(-3,-2)$

And $\dfrac{\gamma+3}{1}=\dfrac{\delta+2}{-1}=\dfrac{-2(-3+2-1)}{1+1}$

$$\Rightarrow \frac{\gamma+3}{1}=\frac{\delta+2}{-1}=2 \Rightarrow \gamma=-1, \delta=-4$$

$R(-1,-4)$

$$M=\left(\frac{1-3}{2}, \frac{2-2}{2}\right)=(-1,0)$$

$$N=\left(\frac{-3-1}{2}, \frac{-2-4}{2}\right)=(-2,-3)$$

$$MN=\sqrt{1+9}=\sqrt{10}$$

Question46

If the slopes of the lines represented by the equation $6x^2+2hxy+4y^2=0$ are in the ratio $2:3$, then the value of h such that

both the lines make acute angles with the positive X-axis measured in positive direction is

Options:

A.

5

B.

$\frac{5}{2}$

C.

-5

D.

$-\frac{5}{2}$

Answer: C

Solution:

$6x^2 + 2hxy + 4y^2 = 0$

Let slopes be m_1 of m_2

$$\therefore \quad \frac{m_1}{m_2} = \frac{2}{3}$$

$$\text{And} \quad m_1 m_2 = \frac{a}{b} = \frac{6}{4}$$

$$m_1 \cdot \frac{3m_1}{2} = \frac{6}{4}$$

$$m_1^2 = 1 \Rightarrow m = \pm 1 \quad (+\text{ ve angle})$$

$$m_1 = 1$$

$$m_2 = \frac{3}{2}$$

$$\therefore \quad m_1 + m_2 = \frac{-2h}{b}$$

$$\Rightarrow \quad 1 + \frac{3}{2} = \frac{-2h}{4}$$

$$\Rightarrow \quad h = -5$$

Question47

If $(3, -2)$ is the centre of the circle $S \equiv x^2 + y^2 + 2gx + 2fy - 23 = 0$ and A is a point on the circle $S = 0$ such that its distance from a point $P(-1, -5)$ is least, then $A =$

Options:

A.

$(3, -2)$

B.

$\left(\frac{9}{5}, \frac{28}{5}\right)$

C.

$\left(\frac{3}{5}, -\frac{2}{5}\right)$

D.

$\left(\frac{-9}{5}, \frac{-28}{5}\right)$

Answer: D

Solution:

Given, equation of circle

$x^2 + y^2 + 2gx + 2fy - 23 = 0$

Centre $O(3, -2) \equiv (-g, -f)$

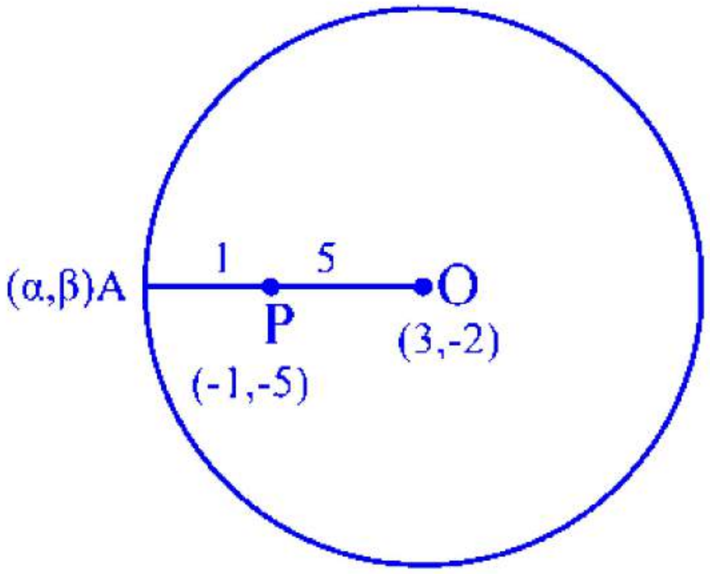
$g = -3, f = 2$

Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{9 + 4 + 23} = 6$

$OP = \sqrt{(3+1)^2 + (-2+5)^2} = \sqrt{16 + 9} = 5$

$\because r > OP$

$\therefore$ Minimum distance



$$\frac{5\alpha+3}{6}=-1 \Rightarrow \alpha=\frac{-9}{5}$$
$$\frac{5\beta-2}{6}=-5 \Rightarrow \beta=\frac{-28}{5}$$
$$\therefore A\left(\frac{-9}{5},\frac{-28}{5}\right)$$

Question48

Two circles which touch both the coordinate axes intersect at the points A and B. If $A=(1,2)$, then $AB=$

Options:

A.

5

B.

13

C.

$2\sqrt{2}$

D.

$\sqrt{2}$

Answer: D

Solution:

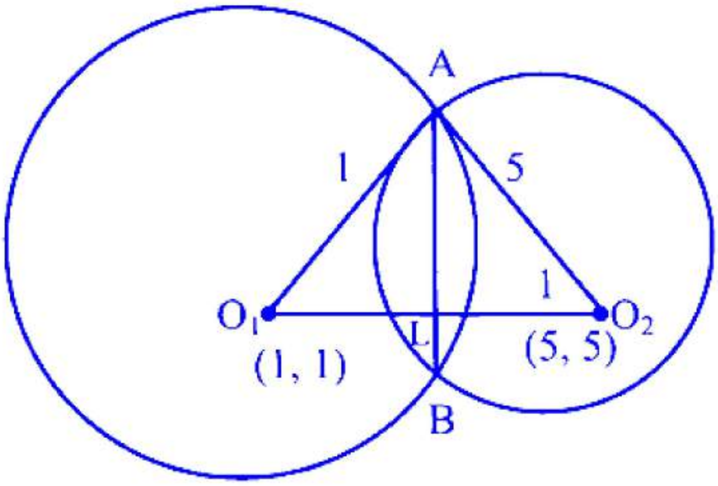
Equation of circle

$(x-a)^2+(y-a)^2=a^2$

Passing through $A(1,2)$

$(1-a)^2+(2-a)^2=a^2$

$a^2-6a+5=0$
$a=1,5$
$S_1 \Rightarrow (x-1)^2+(y-1)^2=1$
$S_2 \Rightarrow (x-5)^2+(y-5)^2=5^2$
Equation of $AB \Rightarrow S_1-S_2=0$
$8x+8y-24-24=-24$



$x+y-3=0$

$L:\left(\frac{5k+1}{k+1},\frac{5k+1}{k+1}\right)$

Lies on AB,

$10k+2-3k-3=0$

$7k=1$

$k=\frac{1}{7}\Rightarrow L=\left(\frac{3}{2},\frac{3}{2}\right)$

$\therefore\quad AB=2AL$

$=2\sqrt{\frac{1}{4}+\frac{1}{4}}=\frac{2}{\sqrt{2}}=\sqrt{2}$

Question49

The lines $4x-3y+2=0$ intersects the circle $x^2+y^2-2x+6y+c=0$ at two points A, B and $AB=8$. If $(1,k)$ is a point on the given circle and $k>0$, then $k=$

Options:

A.

8

B.

4

C.

2

D.

1

Answer: C

Solution:

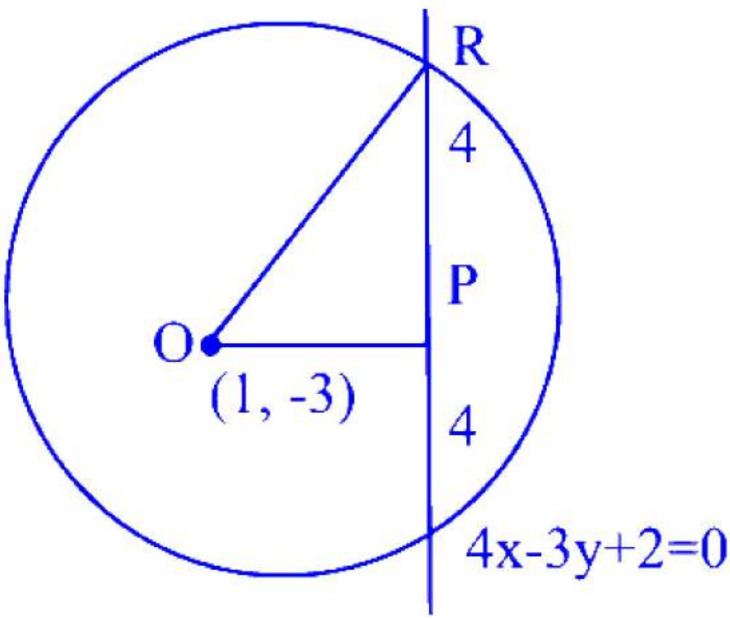
Given, equation of circle

$S: x^2+y^2-2x+6y+c=0$

Centre $O(1,-3)$

$r=\sqrt{1+9-c}=\sqrt{10-c}$

$OP=\left|\frac{4+9+2}{\sqrt{4^2+3^2}}\right|=3$



$\therefore \quad OR=\sqrt{3^2+4^2}=5$

$\Rightarrow \quad \sqrt{10-c}=5 \Rightarrow c=-15$

$\quad S_1(1,k)=1^2+k^2-2+6k-15=0$

$\Rightarrow \quad k^2+6k-16=0$

$\Rightarrow \quad (k+8)(k+2)=0 \Rightarrow k=-8,2$

$\because \quad k>0$

$\therefore \quad k=2$

Question50

If $2x-3y+5=0$ and $4x-5y+7=0$ are the equations of the normals drawn to a circle and $(2,5)$ is a point on the given circle, then the radius of the circle is

Options:

A.

1

B.

2

C.

3

D.

4

Answer: B

Solution:

$(\text{ Normal })_1 = 2x - 3y + 5 = 0 \qquad \ldots(i)$

$(\text{ Normal })_2 = 4x - 5y + 7 = 0 \qquad \ldots(ii)$

Solving Eqs. (i) and (ii), we get

$x = 2, y = 3$

$\because$ Centre $O(2, 3)$

$\therefore$ Radius $= \sqrt{(2-2)^2 + (5-3)^2} = 2$

Question51

If (α, β) is the centre of the circle which passes through the point $(1, -1)$ and cuts the circles

$$x^2 + y^2 + 2x - 3y - 5 = 0, x^2 + y^2 - 3x + 2y + 1 = 0$$

orthogonally, then $\alpha - 5\beta =$

Options:

A.

-10

B.

5

C.

-11

D.

10

Answer: A

Solution:

Given equation of circles

$$S_1 : x^2 + y^2 + 2x - 3y - 5 = 0$$
$$g_1 = 1, f_1 = \frac{-3}{2}, c_1 = -5$$
$$S_2 : x^2 + y^2 - 3x + 2y + 1 = 0$$
$$g_2 = \frac{-3}{2}, f_2 = 1, c_2 = 1$$

Let $S = x^2 + y^2 + 2\alpha x + 2\beta y + c = 0$

$$\begin{aligned}
\therefore \quad & 2gg_1 + 2ff_1 = c_1 + c_2 \\
& 2\alpha - 3\beta = c - 5 && \ldots(i) \\
& 2gg_2 + 2ff_2 = c_1 + c_2 \\
& -3\alpha + 2\beta = c + 1 && \ldots(ii) \\
& (1, -1), 1 + 1 + 2\alpha - 2\beta + c = 0 \\
& 2\alpha - 2\beta = -2 - c && \ldots(iii)
\end{aligned}$$

By Eqs. (i) and (iii), By Eqs. (ii) and (iii),

$$\begin{aligned}
&-\beta = 2c - 3 && -\alpha = -1 \\
&\Rightarrow \beta = 3 - 2c && \Rightarrow \alpha = 1
\end{aligned}$$

By Eqs. (iii),

$$\begin{aligned}
\Rightarrow \quad & 2 - 2\beta = -2 - c \\
\Rightarrow \quad & 2\beta = c + 4 \\
\Rightarrow \quad & 2(3 - 2c) = c + 4 \\
\Rightarrow \quad & 6 - 4c = c + 4 \\
\Rightarrow \quad & 5c = 2 \Rightarrow c = \tfrac{2}{5} \\
\Rightarrow \quad & \beta = 3 - \tfrac{4}{5} = \tfrac{11}{5} \\
& 5\beta = 11 \\
\therefore \quad & \alpha - 5\beta = -10
\end{aligned}$$

Question52

The centre of the circle touching the circles $x^2 + y^2 - 4x - 6y - 12 = 0$

$x^2 + y^2 + 6x + 18y + 26 = 0$ at their point of contact and passing through the point $(1, -1)$ is

Options:

A.

$\left(\frac{1}{3}, -1\right)$

B.

$\left(\frac{1}{5}, \frac{6}{5}\right)$

C.

$\left(\frac{1}{2}, 1\right)$

D.

$\left(-\frac{1}{4}, -\frac{1}{2}\right)$

Answer: A

Solution:

We have two circles given by:

$S_1 : x^2 + y^2 - 4x - 6y - 12 = 0$
$S_2 : x^2 + y^2 + 6x + 18y + 26 = 0$

We need to find a third circle that:

- **Touches both S_1 and S_2 at their point of contact**
- **Passes through the point** $(1, -1)$

Any circle that touches both S_1 and S_2 at their point of contact can be written as:

$\left(x^2 + y^2 - 4x - 6y - 12\right) + \lambda\left(x^2 + y^2 + 6x + 18y + 26\right) = 0$

Since the circle also passes through the point $(1, -1)$, substitute $x = 1, y = -1$ into the equation:
$(1)^2 + (-1)^2 - 4(1) - 6(-1) - 12 + \lambda\left[(1)^2 + (-1)^2 + 6(1) + 18(-1) + 26\right] = 0$

Calculate each term step-by-step:

- First bracket: $1 + 1 - 4 - (-6) - 12 = 2 - 4 + 6 - 12 = -8$ (Notice $-6 \times -1 = +6$)
- Second bracket: $1 + 1 + 6 - 18 + 26 = 2 + 6 - 18 + 26 = 8 - 18 + 26 = -10 + 26 = 16$

So the equation becomes:

$-8 + \lambda(16) = 0$

Solve for λ:

$\lambda = \frac{8}{16} = \frac{1}{2}$

Now plug $\lambda = \frac{1}{2}$ into our general circle equation:

$$\left(x^2+y^2-4x-6y-12\right)+\frac{1}{2}\left(x^2+y^2+6x+18y+26\right)=0$$
$$x^2+y^2-4x-6y-12+\frac{1}{2}x^2+\frac{1}{2}y^2+3x+9y+13=0$$
$$\left(1+\frac{1}{2}\right)x^2+\left(1+\frac{1}{2}\right)y^2+(-4+3)x+(-6+9)y+(-12+13)=0$$
$$\frac{3}{2}x^2+\frac{3}{2}y^2-x+3y+1=0$$

To make it simpler, multiply both sides by 2:

$3x^2+3y^2-2x+6y+2=0$

Divide all terms by 3 to get standard form:

$x^2+y^2-\frac{2}{3}x+2y+\frac{2}{3}=0$

The center of a circle $x^2+y^2+2gx+2fy+c=0$ is $(-g,-f)$.

From the equation:

- $2g=-\frac{2}{3}$ so $g=-\frac{1}{3}$
- $2f=2$ so $f=1$

Therefore, the center is $\left(\frac{1}{3},-1\right)$.

Question53

The number of normals that can be drawn through the point $(2,0)$ to the parabola $y^2=7x$ is

Options:

A.

0

B.

1

C.

2

D.

3

Answer: B

Solution:

$y^2 = 7x$

$4a = 7 \Rightarrow a = \frac{7}{4}$

Point $(2, 0)$

Equation of normal at $(at^2, 2at)$

$$\begin{aligned} \Rightarrow \quad & y + tx = 2at + at^3 \\ \Rightarrow \quad & 0 + 2t = 2at + at^3 \\ \Rightarrow \quad & 2t = \frac{7}{2}t + \frac{7}{4}t^3 \end{aligned}$$

$$\Rightarrow \quad t = 0$$

$$t^2 = \frac{4}{7}\left(2 - \frac{7}{2}\right) = \frac{-6}{7}$$

Not possible.

$\therefore$ Only one normal.

Question54

If m_1 and m_2 are the slopes of the tangents drawn from the point $(1, 4)$ to the parabola $y^2 = 11x$, then $2\left(m_1^2 + m_2^2\right) =$

Options:

A.

24

B.

22

C.

21

D.

18

Answer: C

Solution:

$y^2 = 11x, 4a = 11$

$\therefore$ Equation of tangent

$$y = mx + \frac{a}{m}$$
$$\Rightarrow \quad y = mx + \frac{11}{4m}$$

At (1, 4)

$$4 = m + \frac{11}{4m}$$
$$\Rightarrow \quad 16m = 4m^2 + 11$$
$$\Rightarrow \quad 4m^2 - 16m + 11 = 0$$
$$\Rightarrow \quad m_1 + m_2 = 4$$
$$\Rightarrow \quad m_1 m_2 = \frac{11}{4}$$
$$(m_1 + m_2)^2 = m_1^2 + m_2^2 + 2m_1 m_2$$
$$\Rightarrow \quad 16 - \frac{11}{2} = m_1^2 + m_2^2$$
$$\Rightarrow \quad 21 = 2\left(m_1^2 + m_2^2\right)$$

Question55

If the perpendicular distance from the focus of an ellipse $\frac{x^2}{9} + \frac{y^2}{b^2} = 1 (b < 3)$ to its corresponding directrix is $\frac{4}{\sqrt{5}}$, then the slope of the tangent to this ellipse drawn at $\left(\frac{3}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$ is

Options:

A.

$-\frac{2}{3}$

B.

$\frac{2}{3}$

C.

$\frac{3}{2}$

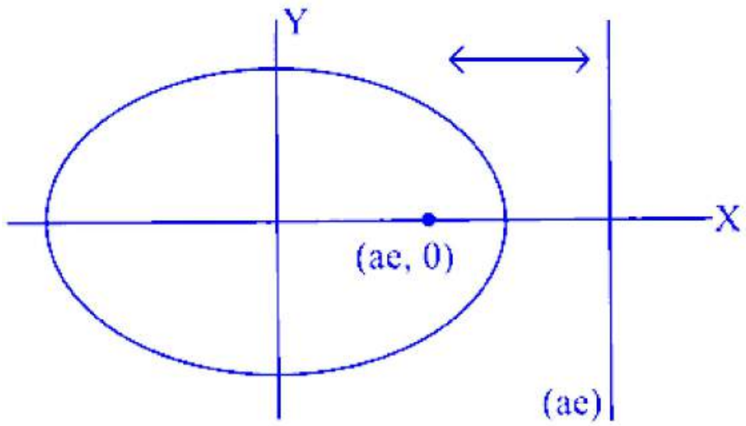
D.

$-\frac{3}{2}$

Answer: A

Solution:

$\frac{x^2}{9} + \frac{y^2}{b^2} = 1$



$\because \frac{a}{e} - ae = \frac{4}{\sqrt{5}}$

$\Rightarrow \frac{3}{e} - 3e = \frac{4}{\sqrt{5}} \Rightarrow 3 - 3e^2 = \frac{4e}{\sqrt{5}}$

$\Rightarrow 3\sqrt{5}e^2 + 4e - 3\sqrt{5} = 0$

$\Rightarrow e = \frac{-4 \pm \sqrt{16 + 180}}{6\sqrt{5}}$

$= \frac{-4 \pm 14}{6\sqrt{5}} = \frac{10}{6\sqrt{5}} = \frac{\sqrt{5}}{3}$

$b^2 = a^2\left(1 - e^2\right)$

$= 9\left(1 - \frac{5}{9}\right) = 4$

Slope of tangent at $\left(\frac{3}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$

$\frac{dy}{dx} = \frac{-2x_1}{9} \cdot \frac{b^2}{2y_1}$

$= \frac{-2 \times \frac{3}{\sqrt{2}} \times 4}{9 \times 2 \times \frac{2}{\sqrt{2}}} = \frac{-2}{3}$

Question56

The length of the chord of the ellipse $\frac{x^2}{4} + y^2 = 1$ formed on the line $y = x + 1$ is

Options:

A.

$2\sqrt{2}$

B.

$\frac{4}{5}\sqrt{2}$

C.

$4\sqrt{2}$

D.

$\frac{8}{5}\sqrt{2}$

Answer: D

Solution:

We have,

$$\frac{x^2}{4}+\frac{y^2}{1}=1 \text{ and } y=x+1$$
$$\Rightarrow \quad \frac{x^2}{4}+(x+1)^2=1$$
$$\Rightarrow \quad \frac{x^2}{4}+x^2+2x=0$$
$$\Rightarrow \quad x=0 \text{ and } \frac{5x}{4}=-2x=\frac{-8}{5}$$
$$y=1, y=\frac{-3}{5}$$
$$\therefore \text{ Length of chord } =\sqrt{\left(\frac{8}{5}\right)^2+\left(\frac{8}{5}\right)^2}=\frac{8}{5}\sqrt{2}$$

Question57

Let P, Q, R, S be the points of intersection of the circle $x^2+y^2=4$ and the hyperbola $xy=\sqrt{3}$. If $P=(\alpha, \beta)$ and $\alpha>\beta>0$, then the equation of the tangent drawn at P to the hyperbola is

Options:

A.

$x+y=2$

B.

$x+\sqrt{3y}=2\sqrt{3}$

C.

$\sqrt{3x}+y=\sqrt{3}$

D.

$x-y=0$

Answer: B

Solution:

$x^2 + y^2 = 4$

$xy = \sqrt{3}$

Point of intersections

$$\begin{aligned} & x^2 + \frac{3}{x^2} = 4 \\ \Rightarrow \quad & x^4 - 4x^2 + 3 = 0 \\ \Rightarrow \quad & \left(x^2 - 3\right)\left(x^2 - 1\right) = 0 \\ \Rightarrow \quad & x = \pm\sqrt{3}, x = \pm 1 \\ & y = \pm 1, y = \pm\sqrt{3} \end{aligned}$$

$\because P(\alpha, \beta) \quad \alpha > \beta > 0$

$\therefore P(\sqrt{3}, 1)$

$\therefore$ Equation of tangent at $P(\sqrt{3}, 1)$ to

$xy = \sqrt{3}$

$$\frac{x(1) + y(\sqrt{3})}{2} = \sqrt{3}$$

$$\Rightarrow \quad x + \sqrt{3}y = 2\sqrt{3}$$

Question58

The number of values of ' k ' for which the points $(-4, 9, k), (-1, 6, k), (0, 7, 10)$ from right-angled isosceles triangle is

Options:

A.

0

B.

1

C.

2

D.

4

Answer: C

Solution:

We have,

$A(-4, 9, k), B(-1, 6, k)$ and $C(0, 7, 10)$

$$\mathbf{AB} = 3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 0\hat{\mathbf{k}}, |\mathbf{AB}| = 3\sqrt{2}$$
$$\mathbf{BC} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + (10 - k)\hat{\mathbf{k}}$$
$$|\mathbf{BC}| = \sqrt{1 + 1 + (10 - k)^2}$$

And $\mathbf{AC} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + (10 - k)\hat{\mathbf{k}}$,

$$|\mathbf{AC}| = \sqrt{16 + 4 + (10 - k)^2}$$
$\therefore |\mathbf{AC}| > |\mathbf{BC}|$
$\therefore AC \rightarrow$ Hypotenuse
$\therefore \mathbf{AB} \cdot \mathbf{BC} = 0$
$\therefore |\mathbf{AB}| = |\mathbf{BC}|$
$\Rightarrow 18 = 2 + (10 - k)^2$
$10 - k = \pm 4 \Rightarrow k = 10 \pm 4 = 14$ or 6

$\therefore$ No. of values of $k = 2$

Question59

A line makes angles $60^{\circ}, 45^{\circ}, \theta$ with positive X, Y, Z axes respectively. If θ is an acute angle, then $\tan \theta =$

Options:

A.

$\sqrt{3}$

B.

$\frac{1}{\sqrt{3}}$

C.

1

D.

2

Answer: A

Solution:

We have,

$$\alpha = 60^\circ, \beta = 45^\circ, \gamma = \theta$$

$\because \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad \cos^2 60 + \cos^2 45^\circ + \cos^2\theta = 1$

$\Rightarrow \quad \cos^2\theta = 1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4}$

$\Rightarrow \quad \cos\theta = \frac{1}{2} \Rightarrow \tan\theta = \sqrt{3}$

Question60

If the foot of the perpendicular drawn from the point $(2, 0, -3)$ to the plane π is $(1, -2, 0)$ and the equation of the plane π is $ax + by - 3z + d = 0$, then $a + b + d =$

Options:

A.

0

B.

1

C.

6

D.

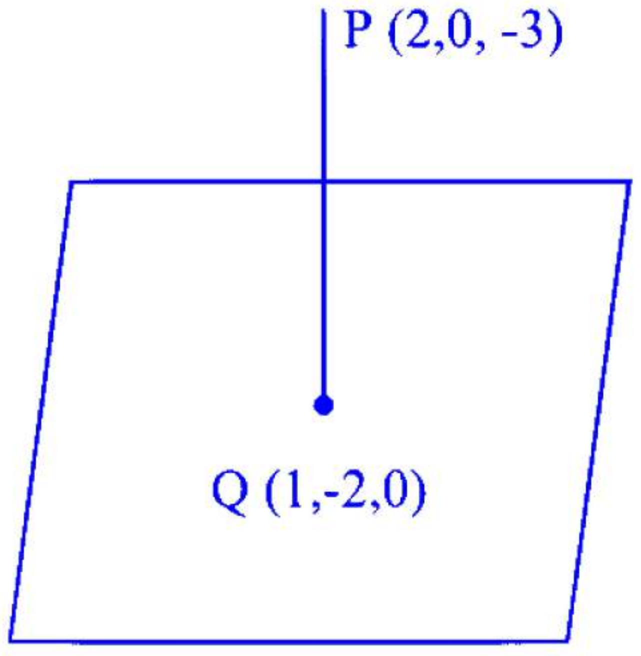
2

Answer: C

Solution:

$$\mathbf{PQ} = -\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

$$\pi : ax + by - 3z + d = 0$$



$\therefore \quad \mathbf{n} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$
$\therefore \quad \mathbf{PQ} \| \mathbf{n}$
$\therefore \quad \frac{-1}{a} = \frac{-2}{b} = \frac{3}{-3} \Rightarrow a = 1, b = 2$
$\therefore Q(1, -2, 0)$ lies on plane
$x + 2y - 3z + d, 1 - 4 + d = 0$
$d = 3$
$a + b + d = 1 + 2 + 3 = 6$

Question61

If $[t]$ represents the greatest integer $\leq t$, then the value of $\lim_{x \to 3} \frac{11 - [2-x]}{[x+10]}$ is

Options:

A.

1

B.

8

C.

5

D.

does not exist

Answer: A

Solution:

$$\lim_{x \to 3} \frac{11-[2-x]}{[x+10]}$$
$$\lim_{x \to 3} = \frac{11-2-[-x]}{[x]+10}$$
$$\text{LHL} = \lim_{x \to 3^-} \frac{11-2-[-x]}{[x]+10} = \frac{9+3}{2+10} = 1$$
$$\text{RHL} = \lim_{x \to 3^+} \frac{9-[-x]}{[x]+10} = \frac{9+4}{3+10} = 1$$
$$\therefore \text{ limiting value } = 1$$

Question62

If the real valued function

$$f(x) = \begin{cases} \frac{\cos 3x - \cos x}{x \sin x}, & \text{if} \quad x < 0 \\ p, & \text{if} \quad x = 0 \\ \frac{\log(1+q \sin x)}{x}, & \text{if} \quad x > 0 \end{cases}$$

is continuous at $x = 0$, then $p + q =$

Options:

A.

4

B.

-4

C.

8

D.

-8

Answer: D

Solution:

$$f(x)=\begin{cases}\frac{\cos 3x-\cos x}{x\sin x}, & x<0\\ p, & x=0\\ \frac{\log(1+q\sin x)}{x}, & x>0\end{cases}$$

$\because f(x)$ is continuous at $x=0$

$f\left(0^{-}\right)=f(0)=f\left(0^{+}\right)$

$$\lim_{x\to 0^{-}}\frac{\cos 3x-\cos x}{x\sin x}=p$$

$$=\lim_{x\to 0^{+}}\frac{\log(1+q\sin x)}{x}$$

$$\lim_{x\to 0^{-}}\frac{-2\sin 2x\times\sin(x)}{x\sin x}=p$$

$$=\lim_{x\to 0^{+}}\frac{\log(1+q\sin x)(q\sin x)}{x(q\sin x)}$$

$$-4=p=q$$

$\therefore p+q=-8$

Question63

If

$$y=\sqrt{\log\left(x^2+1\right)+\sqrt{\log\left(x^2+1\right)+\sqrt{\log\left(x^2+1\right)+\ldots+\infty}}},\text{, 100.00,}$$

$|x|<1$, **then** $\frac{dy}{dx}=$

Options:

A.

$\frac{x^2+1}{2y-1}$

B.

$\frac{2x}{2y-1}$

C.

$\frac{1}{(x^2+1)(2y-1)}$

D.

$\frac{2x}{(x^2+1)(2y-1)}$

Answer: D

Solution:

$$y=\sqrt{\log\left(x^2+1\right)+\sqrt{\log\left(x^2+1\right)+\sqrt{\log\left(x^2+1\right)}}}\ldots\infty$$

$$\Rightarrow y=\sqrt{\log\left(x^2+1\right)+y}$$
$$\Rightarrow y^2=\log\left(x^2+1\right)+y$$
$$\Rightarrow y^2-y=\log\left(x^2+1\right)$$

Differentiating wrt x, we get

$$(2y-1)\frac{dy}{dx}=\frac{2x}{x^2+1}$$
$$\frac{dy}{dx}=\frac{2x}{(2y-1)\left(x^2+1\right)}$$

Question64

If $x=\sqrt{1-\tan y}$, then $\frac{dy}{dx}=$

Options:

A.

$\frac{2x}{x^4+2x^2+2}$

B.

$-\frac{2x}{x^4-2x^2+2}$

C.

$\frac{2x}{x^4-2x^2+2}$

D.

$-\frac{2x}{x^4+2x^2+2}$

Answer: B

Solution:

$x=\sqrt{1-\tan y}$

$\tan y=1-x^2$

Differentiating w.r.t x, we get

$$\sec^2 y\frac{dy}{dx}=-2x$$
$$\Rightarrow\quad \frac{dy}{dx}=\frac{-2x}{\sec^2 y}=\frac{-2x}{1+\left(1-x^2\right)^2}$$
$$=\frac{-2x}{x^4-2x^2+2}$$

Question65

If $y=\sec^{-1}x$, then $\frac{d^2y}{dx^2}=$

Options:

A.

$\frac{1-2x^2}{x|x|(x^2-1)^{\frac{3}{2}}}$

B.

$\frac{1-x^2}{x^2(x^2-1)^{\frac{3}{2}}}$

C.

$\frac{1-x^2}{-x^2(x^2-1)^{\frac{3}{2}}}$

D.

$\frac{1+2x^2}{x|x|(x^2-1)^{\frac{3}{2}}}$

Answer: A

Solution:

$$y=\sec^{-1}x$$
$$x=\sec y, \cos y=\frac{1}{x}, \sin y=\frac{\sqrt{x^2-1}}{x}$$
$$1=\sec y\tan y\frac{dy}{dx}$$
$$\Rightarrow\frac{dy}{dx}=\cot y\cdot\cos y$$
$$\Rightarrow\frac{d^2y}{dx^2}=\left[\cot y\cdot(-\sin y)+\cos y\left(-\operatorname{cosec}^2 y\right)\right]\frac{dy}{dx}$$
$$=-\cos y\left(1+\frac{1}{\sin^2 y}\right)\cdot\cot y\cdot\cos y$$
$$=-\cos^3 y\frac{\left(1+\sin^2 y\right)}{\sin^3 y}=\frac{1}{x|x|}\frac{\left(1-2x^2\right)}{\left(x^2-1\right)^{\frac{3}{2}}}$$

Question66

If $x=\sin 2\theta\cos 3\theta, y=\sin 3\theta\cos 2\theta$, then $\frac{dy}{dx}=$

Options:

A.

$\frac{2\cos 5\theta+\sin 3\theta \sin 2\theta}{2\cos 5\theta-\cos 3\theta \cos 2\theta}$

B.

$\frac{2\cos 5\theta-\sin 3\theta \sin 2\theta}{2\cos 5\theta+\cos 3\theta \cos 2\theta}$

C.

$\frac{2\cos 5\theta+\cos 3\theta \cos 2\theta}{2\cos 5\theta-\sin 3\theta \sin 2\theta}$

D.

$\frac{2\cos 5\theta-\sin 3\theta \sin 2\theta}{2\cos 5\theta-\cos 3\theta \cos 2\theta}$

Answer: C

Solution:

$$\begin{aligned}
& x=\sin 2\theta \cos 3\theta, y=\sin 3\theta \cos 2\theta \\
\Rightarrow\ & 2x=2\sin 2\theta \cos 3\theta, 2y=2\sin 3\theta \cdot \cos 2\theta \\
\Rightarrow\ & 2x=\sin 5\theta-\sin \theta, 2y=\sin 5\theta+\sin \theta \\
& \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{5\cos 5\theta+\cos \theta}{5\cos 5\theta-\cos \theta} \\
& =\frac{4\cos 5\theta+\cos 5\theta+\cos \theta}{4\cos 5\theta+\cos 5\theta-\cos \theta} \\
& =\frac{2\cos 5\theta+\cos 3\theta \cdot \cos 2\theta}{2\cos 5\theta-\sin 3\theta \cdot \sin 2\theta}
\end{aligned}$$

Question67

If the tangent and the normal drawn to the curve $xy^2+x^2y=12$ at the point $(1,3)$ meet the X -axis in T and N respectively, then $TN=$

Options:

A.

$\frac{7}{5}$

B.

$\frac{45}{7}$

C.

$\frac{3\sqrt{274}}{7}$

D.

$\frac{274}{35}$

Answer: D

Solution:

$xy^2 + x^2y = 12$

Differentiating with respect to x,

$$\begin{aligned} & x(2y)\frac{dy}{dx} + y^2 + 2xy + x^2\frac{dy}{dx} = 0 \\ & \text{At}(1,3) \\ & (6)\frac{dy}{dx} + 9 + 6 + \left(\frac{dy}{dx}\right) = 0 \\ \Rightarrow \quad & 7\frac{dy}{dx} = -15 \Rightarrow \left(\frac{dy}{dx}\right) = -\frac{15}{7} \end{aligned}$$

Tangent

$$\begin{aligned} & (y-3) = \frac{-15}{7}(x-1) \\ & T \Rightarrow y = 0 \\ & \Rightarrow x = \frac{7}{5} + 1 \end{aligned}$$

Normal

$$\begin{aligned} & (y-3) = \frac{7}{15}(x-1) \\ & N \Rightarrow y = 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-45}{7} + 1 \\ TN &= \frac{7}{5} + \frac{45}{7} \\ &= \frac{49+225}{35} = \frac{274}{35} \end{aligned}$$

Question68

A man of 5 feet height is walking away from a light fixed at a height of 15 feet at the rate of of K miles/hour. If the rate of increase of his shadow is $\frac{11}{5}$ feet $/\mathrm{sec}$, then $K =$ (Take 1 mile $= 5280$ feet)

Options:

A.

2

B.

3

C.

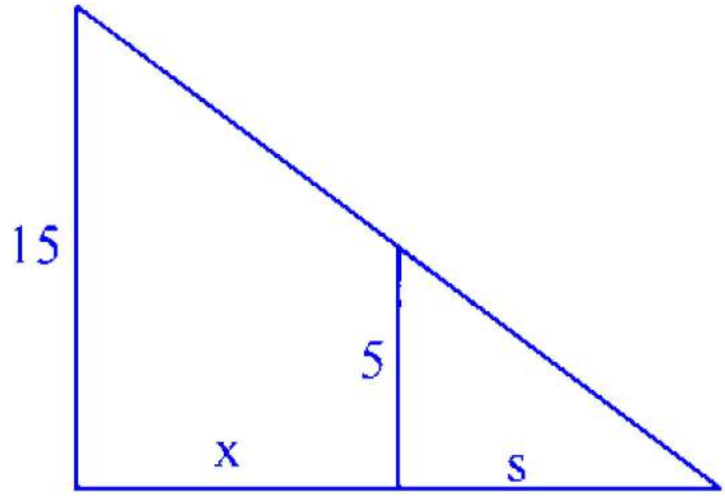
4

D.

5

Answer: B

Solution:



$$\begin{aligned}
&\Rightarrow \quad \frac{15}{5}=\frac{x+s}{s} \\
&\Rightarrow \quad 3s=x+s \\
&\Rightarrow \quad 2s=x \\
&\Rightarrow \quad 2\frac{ds}{dt}=\frac{dx}{dt} \\
&\Rightarrow \quad \frac{ds}{dt}=\frac{11}{5} \\
&\Rightarrow \quad \frac{dx}{dt}=\frac{22}{5}\text{feet/sec}
\end{aligned}$$

$$\begin{aligned}
& =\frac{22}{5}\times\frac{1}{5280}\times 3600\text{ m/hr} \\
& =3\text{mile/hr} \\
k & =3
\end{aligned}$$

Question69

There is a possible error of 0.03 cm in a scale of length 1 foot with which the height of a closed right circular cylinder and the diameter of a sphere are measured as 3.5 feet each. If the radii of both cylinder and sphere are

same, then the approximate error in the sum of the surface areas of both cylinder and sphere is (in square feet)

Options:

A.

0.385

B.

0.0962

C.

0.77

D.

0.1925

Answer: D

Solution:

Finding the radius:

The diameter of the sphere is given as 3.5 feet. The radius is half of the diameter, so $r = \frac{3.5}{2} = 1.75$ ft.

The cylinder's height is also 3.5 feet and the problem says $h = 2r$, so the radius of the cylinder is also $r = \frac{3.5}{2} = 1.75$ ft.

Error conversion:

The possible error in the scale is 0.03 cm, but we need to convert this to feet. Since 1 foot = 30.48 cm: $\Delta r = \frac{0.03}{30.48}$ ft.

Total Surface Area:

The surface area of a sphere is $4\pi r^2$. The surface area of a closed cylinder is $2\pi rh + 2\pi r^2$. So, the total surface area:
$A_{\text{total}} = 4\pi r^2 + 2\pi rh + 2\pi r^2 = 6\pi r^2 + 2\pi rh$ Since $h = 2r$ for the cylinder here,
$A_{\text{total}} = 6\pi r^2 + 2\pi r \times 2r = 6\pi r^2 + 4\pi r^2 = 10\pi r^2$

Finding the Error in Total Surface Area:

The approximate change in surface area, when the radius changes a little by Δr, is: $dA = \frac{dA}{dr} \times \Delta r = 20\pi r \Delta r$

Plug in $r = 1.75$ ft and $\Delta r = \dfrac{0.03}{30.48}$ ft: $dA = 20\pi \times 1.75 \times \frac{0.03}{30.48}$

Now, calculate step-by-step:

- $20 \times 1.75 = 35$
- $\pi \approx \dfrac{22}{7}$
- So, $dA = 35 \times \dfrac{22}{7} \times \dfrac{0.03}{30.48}$

Calculate $35 \times \frac{22}{7} = 110$. So, $dA = 110 \times \frac{0.03}{30.48}$

Multiply $110 \times 0.03 = 3.3$. So, $dA = \frac{3.3}{30.48} \approx 0.108$ (rounded)

Using more exact π gives about 0.1925 square feet as in the answer.

Final approximate error in the sum of surface areas = 0.1925 sq ft.

Question70

For a real number ' a ', if a real valued function $f(x) = 4x^3 + ax^2 + 3x - 2$ is monotonic in its domain, then the range of ' a ' is

Options:

A.

$(-6, 6)$

B.

Empty set

C.

$(-2, 2)$

D.

$(2, 4)$

Answer: A

Solution:

We have,

$$f(x) = 4x^3 + ax^2 + 3x - 2$$
$$f'(x) = 12x^2 + 2ax + 3$$

For monotonic $f'(x) > 0$

$$12x^2 + 2ax + 3 > 0$$
$$D < 0$$
$$\Rightarrow \quad (2a)^2 - 4(12)(3) < 0 \Rightarrow a^2 - 36 < 0$$
$$\Rightarrow \quad (a-6)(a+6) < 0 \Rightarrow a \in (-6, 6)$$

Question71

If the point $P(x_1, y_1)$ lying on the curve $y = x^2 - x + 1$ is the closest point to the line $y = x - 3$, then the perpendicular distance from P to the line $3x + 4y - 2 = 0$ is

Options:

A.

16/5

B.

4

C.

1

D.

7/5

Answer: C

Solution:

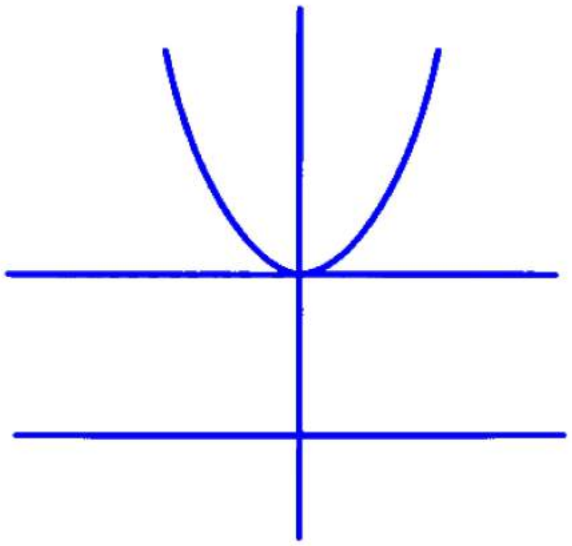
$P(x_1, y_1)$ lies on curve $y = x^2 - x + 1$

$\therefore$ Line $L : y = x - 3$

$\because L$ closest to curve, $y = x^2 - x + 1$

$\therefore$ It lies on common normal

$\therefore m_N = -1 \Rightarrow m_T = 1$



$$
\begin{aligned}
&\frac{dy}{dx} = 2x - 1 \\
&1 = 2x - 1 \Rightarrow x = 1 \\
\Rightarrow \quad & y = 1 - 1 + 1 = 1 \\
\text{At} \quad & P(1,1) L_2 = 3x + 4y - 2 \\
\therefore \quad & D = \left|\frac{3+4-2}{5}\right| = 1
\end{aligned}
$$

Question72

$\int \frac{3^x(x \log 3 - 1)}{x^2} dx =$

Options:

A.

$x \cdot 3^x + C$

B.

$\frac{3^x}{x^2} + C$

C.

$x^2 3^x + C$

D.

$\frac{3^x}{x} + C$

Answer: D

Solution:

Let's call the integral I.

$I = \int \frac{3^x(x\log 3-1)}{x^2} dx$

Let's make a substitution. Set $t = \frac{3^x}{x}$.

Now, calculate the derivative of t with respect to x:

$\frac{dt}{dx} = \frac{3^x \log 3 \cdot x - 3^x}{x^2} = \frac{3^x(x\log 3-1)}{x^2}$

So, $dt = \frac{3^x(x\log 3-1)}{x^2} dx$.

This shows that the part inside our integral, $\frac{3^x(x\log 3-1)}{x^2} dx$, is simply dt.

So the integral becomes $I = \int dt = t + C$.

Recall $t = \frac{3^x}{x}$, so the answer is $I = \frac{3^x}{x} + C$.

Question73

If $\frac{5\pi}{4} < x < \frac{7\pi}{4}$, then $\int \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} dx =$

Options:

A.

$-\sec^2\left(\frac{\pi}{4} - x\right) + C$

B.

$-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + C$

C.

$\sec^2\left(\frac{\pi}{4} - x\right) + C$

D.

$\log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + C$

Answer: D

Solution:

$$\because \frac{5\pi}{4} < x < \frac{7\pi}{4}$$

$$\int \sqrt{\frac{1 - \sin 2x}{1 + \sin 2x}} dx$$

$$= \int \sqrt{\frac{1 - \frac{2\tan x}{1+\tan^2 x}}{1 + \frac{2\tan^2}{1+\tan^2 x}}} dx$$

$$= \int \frac{\sqrt{(\tan x - 1)^2}}{\sqrt{(\tan x + 1)^1}} dx$$

$$= -\int \frac{1 - \tan x}{1 + \tan x} dx$$

$$= -\int \tan\left(\frac{\pi}{4} - x\right) dx + C$$

$$= \frac{-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|}{-1} + C$$

$$= \log\left|\sec\left(\frac{\pi}{4} - x\right)\right| + C$$

Question74

$$\int x \tan^{-1} \sqrt{\frac{1+x^2}{1-x^2}} dx =$$

Options:

A.

$\frac{x^2}{4}\left(\pi - \cos^{-1} x^2\right) + \frac{1}{4}\sqrt{1 - x^2} + C$

B.

$\frac{x^2}{4}\left(\pi - \cos^{-1} x^2\right) + \frac{1}{4}\sqrt{1 - x^4} + C$

C.

$\frac{x^2}{4}\left(\pi + \cos^{-1} x^2\right) - \frac{1}{4}\sqrt{1 - x^4} + C$

D.

$\frac{x^2}{4}\left(\pi + \cos^{-1} x^2\right) - \frac{1}{4}\sqrt{1 - x^2} + C$

Answer: B

Solution:

$$I=\int x\tan^{-1}\sqrt{\frac{1+x^2}{1-x^2}}dx$$

$x^2=\cos\theta \Rightarrow 2xdx=-\sin\theta d\theta$

$$\Rightarrow I=\int \tan^{-1}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}\cdot\left(\frac{-\sin\theta}{2}\right)d\theta\Bigg)$$

$$=\frac{1}{2}\int\tan^{-1}\left(\cot\frac{\theta}{2}\right)(-\sin\theta)d\theta$$

$$=\frac{1}{2}\int\left(\frac{\pi}{2}-\frac{\theta}{2}\right)(-\sin\theta)d\theta$$

$$=\frac{1}{2}\int\left(\frac{\theta}{2}\sin\theta-\frac{\pi}{2}\sin\theta\right)d\theta$$

$$=\frac{\theta}{4}(-\cos\theta)+\frac{\sin\theta}{4}+\frac{\pi}{4}\cos\theta+C$$

$$=\frac{-x^2}{4}\cos^{-1}x^2+\frac{\sqrt{1-x^4}}{4}+\frac{\pi}{4}x^2+C$$

$$=\frac{x^2}{4}\left(\pi-\cos^{-1}x^2\right)+\frac{1}{4}\sqrt{1-x^4}+C$$

Question75

$$\int\frac{1}{(2\cos x+\sin x)^2}dx=$$

Options:

A.

$\frac{1}{2+\tan x}+C$

B.

$-\frac{1}{2\tan x+1}+C$

C.

$\frac{\cos x}{\cos x+2\sin x}+C$

D.

$-\frac{\cos x}{2\cos x+\sin x}+C$

Answer: D

Solution:

$$I=\int \frac{dx}{(2\cos x+\sin x)^2}$$
$$I=\int \frac{\sec^2 x}{(2+\tan x)^2}$$
Put $\tan x+2=t$

$\sec^2 x dx=dt$
$$I=\int \frac{dt}{t^2}$$
$$=\frac{-1}{t}+c=\frac{-1}{\tan x+2}+C$$
$$=\frac{-\cos x}{\sin x+2\cos x}+C$$

Question76

$\int_{-1}^{1} \frac{\log 2-\log(1+x)}{\sqrt{1-x^2}} dx =$

Options:

A.

$\frac{\pi}{8}\log 2$

B.

$-\frac{\pi}{2}\log 2$

C.

$-\frac{\pi}{4}\log 2$

D.

$2\pi\log 2$

Answer: D

Solution:

$$I=\int_{-1}^{1}\frac{\log 2-\log(1+x)}{\sqrt{1-x^2}}dx$$
$$\Rightarrow I=\int_{-1}^{1}\frac{\log 2}{\sqrt{1-x^2}}dx-\int_{-1}^{1}\frac{\log(1+x)}{\sqrt{1-x^2}}dx$$
$$\Rightarrow I=\left[\log 2\sin^{-1}x\right]_{-1}^{1}-\int\frac{\log(1+x)}{\sqrt{1-x^2}}dx$$
$$=\pi\log 2-I_1$$
$$I_1=\int_{-1}^{1}\frac{\log(1+x)}{\sqrt{1-x^2}}dx$$
Put $\begin{array}{c}x=\sin\theta\\ dx=\cos\theta d\theta\end{array}$

$$\Rightarrow I_1=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{\log(1+\sin\theta)}{\cos\theta}\cos\theta d\theta$$
$$\Rightarrow I_1=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\log(1+\sin\theta)d\theta \qquad \ldots(i)$$
King's property,
$$\Rightarrow \quad I_1=\int_{\frac{-\pi}{2}}^{\pi/2}\log(1-\sin\theta)d\theta \qquad \ldots(ii)$$
$$\Rightarrow \quad 2I_1=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\log\left(1-\sin^2\theta\right)d\theta$$
$$\Rightarrow \quad I_1=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\log\cos\theta d\theta$$
$$=2\int_{0}^{\frac{\pi}{2}}\log\cos\theta d\theta$$
$$=2\left(\frac{-\pi}{2}\log 2\right)$$
$$=-\pi\log 2$$
$$\therefore \quad I=2\pi\log 2$$

Question77

$$\int_0^{\frac{\pi}{4}}\frac{\sec x}{3\cos x+4\sin x}dx=$$

Options:

A.

$\log\left(\frac{7}{3}\right)$

B.

$\frac{1}{4}\log\left(\frac{7}{3}\right)$

C.

$\frac{1}{4}\log 7$

D.

$\log 7$

Answer: B

Solution:

$$I=\int_0^{\frac{\pi}{4}}\frac{\sec x}{3\cos x+4\sin x}dx$$
$$=\int_0^{\frac{\pi}{4}}\frac{\sec^2 x}{3+4\tan x}dx$$

Put $\tan x=t\Rightarrow\sec^2 x dx=dt$

$$\Rightarrow\quad I=\int_0^1\frac{dt}{3+4t}$$
$$=\frac{1}{4}[\log 4t+3]_0^1=\frac{1}{4}\log\frac{7}{3}$$

Question78

$\int_{-2}^{4}\left|2-x^2\right|dx=$

Options:

A.

$\frac{8\sqrt{2}}{3}-3$

B.

$\frac{16\sqrt{2}}{3}+12$

C.

$\frac{16\sqrt{2}}{3}-3$

D.

$\frac{8\sqrt{2}}{3}+12$

Answer: B

Solution:

$$I=\int_{-2}^{4}\left|\left(2-x^2\right)\right|dx$$

$$=\int_{-2}^{-\sqrt{2}}\left(x^2-2\right)dx+\int_{-\sqrt{2}}^{+\sqrt{2}}\left(2-x^2\right)dx+\int_{\sqrt{2}}^{+4}\left(x^2-2\right)dx$$

$$=\left[\frac{x^3}{3}-2x\right]_{-2}^{-\sqrt{2}}+\left[2x-\frac{x^3}{3}\right]_{-\sqrt{2}}^{\sqrt{2}}+\left[\frac{x^3}{3}-2x\right]_{\sqrt{2}}^{4}$$

$$\Rightarrow \frac{-2\sqrt{2}}{3}+2\sqrt{2}\Big)-\left(\frac{-8}{3}+4\right)$$

$$+\left[\left(2\sqrt{2}-\frac{2\sqrt{2}}{3}\right)-\left(-2\sqrt{2}+\frac{2\sqrt{2}}{3}\right)\right]$$

$$+\left[\left(\frac{64}{3}-8\right)-\left(\frac{2\sqrt{2}}{3}-2\sqrt{2}\right)\right]$$

$$\Rightarrow \frac{4\sqrt{2}}{3}-\frac{4}{3}+4\sqrt{2}-\frac{4\sqrt{2}}{3}+\frac{40}{3}+\frac{4\sqrt{2}}{3}$$

$$=12+\frac{16\sqrt{2}}{3}$$

Question79

The general solution of the differential equation $\frac{dy}{dx}+(\sec x \operatorname{cosec} x)y=\cos^2 x$

Options:

A.

$y\sec^2 x=\sin^2 x+C$

B.

$y\sec^2 x=\tan x+C$

C.

$y\tan x=\sin x\cos x+C$

D.

$2y\tan x=\sin^2 x+C$

Answer: D

Solution:

We have,

$$\frac{dy}{dx}+(\sec x\cdot\operatorname{cosec} x)y=\cos^2 x$$

$$\text{IF }=\mathrm{e}^{\int\sec x\operatorname{cosec} x dx}$$

$$=e^{\int\frac{2}{\sin 2x}dx}=e^{2\int\operatorname{cosec} 2x dx}$$
$$=e^{2\frac{\ln|\operatorname{cosec} 2x-\cot 2x|}{2}}$$
$$=(\operatorname{cosec} 2x-\cot 2x)$$
$$=\frac{1-\cos 2x}{\sin 2x}=\frac{2\sin^2 x}{2\sin x\cos x}$$
$$=\frac{\sin x}{\cos x}$$

Solution

$$y(\mathrm{IF})=\int\cos^2 x(\mathrm{IF})dx+k$$
$$\Rightarrow y\tan x=\int\sin x\cos x dx+k$$
$$\Rightarrow y\tan x=\frac{\sin^2 x}{2}+k$$
$$\Rightarrow 2y\tan x=\sin^2 x+C$$

Question80

If the differential equation having $y=Ae^x+B\sin x$ as its general solution is $f(x)\frac{d^2y}{dx^2}+g(x)\frac{dy}{dx}+h(x)y=0$, then $f(x)+g(x)+h(x)=$

Options:

A.

$2\cos x$

B.

$4\sin x$

C.

0

D.

$\cos x-\sin x$

Answer: C

Solution:

$$y = Ae^x + B\sin x \qquad \ldots(i)$$
$$\frac{dy}{dx} = Ae^x + B\cos x \qquad \ldots(ii)$$
$$\frac{d^2y}{dx^2} = Ae^x - B\sin x \qquad \ldots(iii)$$

From Eqs. (i) and (iii),

$\frac{1}{2}\left(y + \frac{d^2y}{dx^2}\right) = Ae^x$

From Eqs. (i) and (ii),

$B = \left(y - \frac{dy}{dx}\right)\frac{1}{\sin x - \cos x}$

∴ From Eq. (iii),

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{1}{2}\left(y + \frac{d^2y}{dx^2}\right) - \left(y - \frac{dy}{dx}\right)\frac{\sin x}{\sin x - \cos x}$

$$\Rightarrow \frac{1}{2}\frac{d^2y}{dx^2} = \frac{y}{2} - \frac{\sin x}{\sin x - \cos x}\left(y - \frac{dy}{dx}\right)$$

$\Rightarrow (\sin x - \cos x)\frac{d^2y}{dx^2} = (\sin x - \cos x)y$

$-2\sin x \cdot y + 2\sin x\frac{dy}{dx}$

$$\Rightarrow (\sin x - \cos x)\frac{d^2y}{dx^2} - 2\sin x\frac{dy}{dx} + (2\sin x - \sin x + \cos x)y = 0$$

$$\Rightarrow (\sin x - \cos x)\frac{d^2y}{dx^2} - 2\sin x\frac{dy}{dx} + (\sin x + \cos x)y = 0$$

$\therefore f(x) + g(x) + h(x)$

$= \sin x - \cos x - 2\sin x + \sin x + \cos x$

$= 0$

Chemistry

Question1

In the atomic spectrum of hydrogen, the wavelengths of the spectral lines corresponding to electronic transitions (i) $n = 4$ to $n = 2$ and (ii) $n = 3$ to $n = 1$ are λ_1 and λ_2 Å respectively. The value of ($\lambda_1 - \lambda_2$) (in cm) is (R_{H} = Rydberg constant)

Options:

A.

$\frac{1}{R_{\mathrm{H}}}\left[\frac{24}{101}\right]$

B.

$R_{\mathrm{H}}\left[\frac{24}{101}\right]$

C.

$\frac{1}{R_{\mathrm{H}}}\left[\frac{101}{24}\right]$

D.

$R_{\mathrm{H}}\left[\frac{101}{24}\right]$

Answer: C

Solution:

In the atomic spectrum of hydrogen, the wavenumber corresponding to an electronic transition is given by the Rydberg formula.

$v=R_H\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

For transition (i) : $n=4 \rightarrow n=2$ the corresponding wavelength is λ_1.

For transition (ii) : $n=3 \rightarrow n=1$, the corresponding wavelength is λ_2.

Using the Rydberg formula

$$\frac{1}{\lambda_1}=R_H\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=R_H\left(\frac{1}{4}-\frac{1}{16}\right)$$
$$=R_H\left(\frac{3}{16}\right)$$
$$\frac{1}{\lambda_2}=R_H\left(\frac{1}{1^2}-\frac{1}{3^2}\right)=R_H\left(1-\frac{1}{9}\right)=R_H\left(\frac{8}{9}\right)$$

We now calculate the difference in wavelengths

$$\lambda_1-\lambda_2=\frac{1}{v_1}-\frac{1}{v_2}$$
$$=\frac{1}{\left(\frac{3R_H}{16}\right)}-\frac{1}{\left(\frac{8R_H}{9}\right)}=\frac{16}{3R_H}-\frac{9}{8R_H}$$

Taking LCM,

$$\lambda_1-\lambda_2=\frac{(16.8-9.3)}{24R_H}$$
$$=\frac{(128-27)}{24R_H}=\frac{101}{24R_H}$$

Thus, $\lambda_1-\lambda_2=\frac{1}{R_H}\left(\frac{101}{24}\right)$

Therefore, the correct option is (c).

Question2

Work functions of four metals M_1, M_2, M_3 and M_4 are $4.8, 4.3, 4.75$ and 3.75 eV respectively. The metals which do not show photoelectric effect when light of wavelength 310 nm falls on the metals are

Options:

A.

M_1, M_2 only

B.

M_1, M_3 only

C.

M_1, M_2, M_3 only

D.

M_1, M_2, M_4 only

Answer: C

Solution:

Energy of photon

$E = \frac{1240}{\lambda(\text{in nm})} = \frac{1240}{310} = 4.0\text{eV}$

Given work functions

- $M_1 = 4.8\text{eV}$
- $M_2 = 4.3\text{eV}$
- $M_3 = 4.75\text{eV}$
- $\text{M}_4 = 3.75\text{eV}$

Only metals with $\phi \leq 4.0\text{eV}$ will emit electrons.

M_1, M_2 and M_3 have work functions greater than 4.0 eV thus, do not show no photoelectric effect.

Question3

In second period of the modern periodic table, two elements X and Y have higher first ionisation enthalpy values than the preceding and succeeding elements. X and Y are respectively

Options:

A.

B, C

B.

Al, S

C.

Be, N

D.

Na, S

Answer: C

Solution:

The first ionisation enthalpy generally increases across a period due to increasing effective nuclear charge. However, exceptions to this trend occur when a stable electronic configuration is involved.

In the second period (Li to Ne), two such exceptions occur.

1. Beryllium (Be) The electronic configuration is $1s^2s^2$. The first ionisation enthalpy of beryllium is higher than that of boron, the succeeding element. It is also higher than that of proceeding Ind element is lithium. This is due to the stable, completely filled 2 s subshell, which makes it more difficult to remove an electron.

2.Nitrogen (N) The electronic configuration is $1s^2s^2y^3$. The first ionisation enthalpy of nitrogen is higher than that of oxygen, the succeeding element. It is also higher than the proceeding element is carbon. This is because of the extra stability associated with the half-filled 2p subshell, which resists the removal of an electron.

Therefore, the two elements X and Y are beryllium (Be) and nitrogen (N).

Question4

Consider the following pairs of elements and identify the pairs of elements which have nearly same atomic radius.

I. Y, La

II. Zr, Hf

III. Mo, W

IV. Cr, Mo

Options:

A.

I and II

B.

II and III

C.

III and IV

D.

I and III

Answer: B

Solution:

Atomic radius generally increases down a group. However, due to the lanthanoid contraction, elements of the $5d$ series have nearly the same atomic radii as their counterparts in the $4d$ series. Lanthanoid contraction is the poor shielding effect of the $4f$ electrons, which increases the effective nuclear charge and pulls the valence electrons closer to the nucleus.

- $\mathrm{Zr}(4d)$ and Hf ($5d$) are in the same group, and size of Hf is reduced by the lanthanoid contraction, making their radii nearly identical.
- Mo (4d) and W (5d) are also a pair in the same group where the size of W is reduced by the lanthanoid contraction, resulting in almost identical radii.
- The other pairs, Y-La and Cr-Mo, do not exhibit this effect to the same extent, and their atomic radii differ significantly.

Therefore, the pairs with nearly the same atomic radius are $\mathrm{Zr}-\mathrm{Hf}$ and $\mathrm{Mo}-\mathrm{W}$.

Question5

If the sum of bond orders of O_2^- and O_2^{2-} is x, then bond order of O_2^{2+} will be

Options:

A.

$1.20x$

B.

$1.33x$

C.

$1.50x$

D.

$2.50x$

Answer: A

Solution:

Step 1: Find bond orders using the formula

Bond order = $\frac{1}{2}$(number of bonding electrons - number of antibonding electrons)

Bond order of O_2^- (17 electrons):

$= \frac{1}{2}(10 - 7) = 1.5$

Bond order of O_2^{2-} (18 electrons):

$= \frac{1}{2}(10 - 8) = 1.0$

Bond order of O_2^{2+} (14 electrons):

$= \frac{1}{2}(10 - 4) = 3.0$

Step 2: Add the bond orders for O_2^- and O_2^{2-}

The question says x is their sum.

$x = 1.5 + 1.0 = 2.5$

Step 3: Express the bond order of O_2^{2+} in terms of x

We already found bond order of O_2^{2+} is 3.0.

We want to write 3.0 as a multiple of x:

$$3.0 = k \cdot x$$
$$3.0 = k \cdot 2.5$$
$$k = \frac{3.0}{2.5} = 1.20$$

So, bond order of O_2^{2+} is $1.20x$.

Question6

Identify the molecule / ion in which the ratio of σ to π-bonds is $3 : 2$

Options:

A.

HCO_3^-

B.

$CH_2(CN)_2$

C.

$HClO_4$

D.

XeO_3

Answer: B

Solution:

The structure of $CH_2(CN)_2$ is as follows

So, number of σ bonds $= 6$

Number of π bonds $= 4$

So, the ratio of $\sigma : \pi$ bonds $= 3 : 2$

Question7

At 298 K , a flask ' A ' of unknown volume (V) contains oxygen at 5 atm . Another flask ' B ' of volume 2 L contains helium at 3 atm . Two flasks are connected together by a small tube of zero volume. After the two gases are completely mixed, if the resulting mixture is found to have the mole fraction of oxygen as 0.2 , the volume of flask ' A ' (in L) is

(Assume oxygen and helium as ideal gases)

Options:

A.

0.1

B.

0.3

C.

0.2

D.

0.4

Answer: B

Solution:

Using the ideal gas law, $n=\frac{pV}{RT}$.

Since the temperature and R are constant, they can be treated as a single term.

- Moles of oxygen in flask

$A: n_{O_2}=\frac{5\times V}{RT}$

- Moles of helium in flask

$B: n_{Hc}=\frac{3\times 2}{RT}=\frac{6}{RT}$

- Total moles after mixing:

$n_{\text{total}} \quad = n_{O_2}+n_{He}=\frac{5V+6}{RT}$

The mole fraction of oxygen in the mixture is given as 0.2 .

$x_{O_2}=\frac{n_{O_2}}{n_{\text{total}}}=\frac{5V/RT}{(5V+6)/RT}=0.2$

The RT terms cancel out, leaving

$$\frac{5V}{5V+6}=0.2$$
$$5V=0.2(5V+6)$$
$$5V=V+1.2 \Rightarrow 4V=1.2$$
$$V=0.3\ \mathrm{L}$$

Question8

In which of the following, oxidation state of nitrogen is lowest?

Options:

A.

NH_2OH

B.

NH_4Cl

C.

N_2H_4

D.

HNO_2

Answer: B

Solution:

We need to find out where nitrogen has the lowest (most negative) oxidation state among the options. To do this, we will calculate the oxidation state of nitrogen in each compound.

(a) In NH_2OH (Hydroxylamine):

Let nitrogen's oxidation state be x. Hydrogen (H) is +1 and oxygen (O) is -2.

There are 2 H atoms, so $2 \times (+1) = +2$. There is 1 O atom, so (-2). There is also 1 extra H (in OH), so another $+1$.

$$\begin{aligned} x + 2(+1) + (-2) + (+1) &= 0 \\ x + 1 &= 0 \\ x &= -1 \end{aligned}$$

So, in hydroxylamine, nitrogen is at -1.

(b) In NH_4Cl (Ammonium chloride):

This compound has an ammonium ion (NH_4^+) and a chloride ion (Cl^-).

Let nitrogen's oxidation state be x. Hydrogen is +1, chlorine is -1.

There are 4 H atoms, so $4 \times (+1) = +4$.

$$\begin{aligned} x + 4 \times 1 - 1 &= 0 \\ x + 4(+1) &= +1 \\ x + 4 &= 1 \\ x &= -3 \end{aligned}$$

So, in ammonium chloride, nitrogen is at -3.

(c) In N_2H_4 (Hydrazine):

Let nitrogen's oxidation state be x. Hydrogen is +1.

There are 2 nitrogen and 4 hydrogen atoms.

$$\begin{aligned} 2x + 4(+1) &= 0 \\ 2x &= -4 \\ x &= -2 \end{aligned}$$

So, in hydrazine, nitrogen is at -2.

(d) In HNO_2 (Nitrous acid):

Let nitrogen's oxidation state be x. Hydrogen is +1, oxygen is -2.

There is 1 H and 2 O.

$$\begin{aligned} (+1) + x + 2(-2) &= 0 \\ 1 + x - 4 &= 0 \\ x - 3 &= 0 \\ x &= +3 \end{aligned}$$

So, in nitrous acid, nitrogen is at +3.

Now, let's compare the results: $-1, \ -3, \ -2, \ +3$. The smallest value is -3. This happens in NH_4Cl (ammonium chloride).

Question9

Which of the following processes are reversible?

I. Vaporisation of a liquid at its boiling point.

II. Expansion of gas into vacuum.

III. Transformation of a solid substance into liquid at its melting point.

IV. Neutralisation of an acid by a base.

Options:

A.

I and III

B.

II and III

C.

II and IV

D.

I and IV

Answer: A

Solution:

A reversible process is one that can be reversed by an infinitesimal change in a condition, allowing the system and surroundings to be restored to their original states. Such process is occur at equilibrium.

I. Vaporisation of a liquid at its boiling point This is a reversible process because the liquid and vapour are in equilibrium at the boiling point.

II. Expansion of gas into vacuum This is an irreversible, spontaneous process as the system is not in equilibrium and cannot be reversed without external work.

III. Transformation of a solid into liquid at its melting point This is a reversible processes as the solid and liquid phases are in equilibrium at the melting point.

IV. Neutralisation of an acid by a base This is an irreversible, spontaneous reaction that proceeds to completion.

Therefore, only processes I and III are reversible.

Question10

At $T(\mathrm{~K})$ in a saturated solution of MgCO_3 and $\mathrm{Ag}_2\mathrm{CO}_3$, if the concentration of Mg^{2+} ion is $3.2 \times 10^{-5}\mathrm{M}$, then the concentration of Ag^{+} ion in the solution will be [Given, $K_{\mathrm{sp}}\left(\mathrm{MgCO}_3\right)=1.6 \times 10^{-6}$ and $K_{\mathrm{sp}}\left(\mathrm{Ag}_2\mathrm{CO}_3\right)=8.0 \times 10^{-12}$ at $T(\mathrm{~K})$]

Options:

A.

$\sqrt{1.3} \times 10^{-7}\mathrm{M}$

B.

$\sqrt{1.5} \times 10^{-6}\mathrm{M}$

C.

$\sqrt{1.6} \times 10^{-6}\mathrm{M}$

D.

$\sqrt{1.6} \times 10^{-5}\mathrm{M}$

Answer: D

Solution:

In the saturated solution, both salts are in equilibrium with their ions. The concentration of the common ion, CO_3^{2-}, links the two equilibria.

Step 1 Find $\left[\mathrm{CO}_3^{2-}\right]$ from the MgCO_3 equilibrium.

$$K_{sp}\left(\mathrm{MgCO}_3\right)=\left[\mathrm{Mg}^{2+}\right]\left[\mathrm{CO}_3^{2-}\right]$$
$$\left[\mathrm{CO}_3^{2-}\right]=\frac{1.6 \times 10^{-6}}{3.2 \times 10^{-5}}=0.05\mathrm{M}$$

Step 2 Find $\left[\mathrm{Ag}^{+}\right]$ from the $\mathrm{Ag}_2\mathrm{CO}_3$ equilibrium,

$$K_{sp}\left(\mathrm{Ag}_2\mathrm{CO}_3\right)=\left[\mathrm{Ag}^{+}\right]^2\left[\mathrm{CO}_3^{2-}\right]$$
$$\left[\mathrm{Ag}^{+}\right]^2=\frac{K_{sp}\left(\mathrm{Ag}_2\mathrm{CO}_3\right)}{\left[\mathrm{CO}_3^{2-}\right]}$$
$$=\frac{8.0 \times 10^{-12}}{0.05}=1.6 \times 10^{-10}$$
$$\left[\mathrm{Ag}^{+}\right]=\sqrt{1.6 \times 10^{-10}}=\sqrt{1.6} \times 10^{-5}\mathrm{M}$$

Question11

Temperature of maximum density of H_2O is y K and D_2O is x K. $(x-y)($ in K$)$ is nearly

Options:

A.

7.0

B.

3.5

C.

4.0

D.

8.5

Answer: A

Solution:

The problem asks for the approximate difference between the temperature of maximum density of heavy water (D_2O) and ordinary water (H_2O).

The temperature of maximum density of H_2O is approximately $3.98^\circ C$.

The temperature of maximum density of D_2O is approximately $11.2^\circ C$.

The difference in temperature in Celsius is

$\Delta T_C = 11.2^\circ C - 3.98^\circ C = 7.22^\circ C$

The problem specifies the temperatures in Kelvin, with y being the temperature for H_2O and x for D_2O. The difference is $(x-y)$ in Kelvin.

The magnitude of a temperature difference is the same whether measured in Celsius or Kelvin.

$\Delta T_K = \Delta T_C$

So, the difference in Kelvin is 7.22 K . The value 7.22 K is closest to the option 7.0 K .

Question12

How many of the following metals give oxides and nitrides when burnt in air?

Be, Na, Mg, Ba, Sr, Li, K

Options:

A.

2

B.

3

C.

4

D.

5

Answer: D

Solution:

The reaction of metals with air, which contains both oxygen and nitrogen, can lead to the formation of both oxides and nitrides. The metals that exhibit this dual reactivity are those with a sufficiently high affinity for nitrogen. Let's analyse the given metals.

- Lithium (Li)

$4Li(s) + O_2(g) \rightarrow 2Li_2O(s)$
$6Li(s) + N_2(g) \rightarrow 2Li_3N(s)$

Lithium forms both an oxide and a nitride.

- Sodium (Na) and potassium (K)

$2Na(s) + O_2(g) \rightarrow Na_2O_2(s)$ (primarily peroxide)

$K(s) + O_2(g) \rightarrow KO_2(s)$ (primarily superoxide)

These alkali metals do not readily form nitrides when burnt in air.

- Beryllium (Be)

$2Be(s) + O_2(g) \rightarrow 2BeO(s)$
$3Be(s) + N_2(g) \rightarrow Be_3N_2(s)$

Beryllium forms both an oxide and a nitride.

Magnesium (Mg)
$2Mg(s) + O_2(g) \rightarrow 2MgO(s)$
$3Mg(s) + N_2(g) \rightarrow Mg_3N_2(s)$

Magnesium forms both an oxide and a nitride.

- Strontium (Sr)

$2Sr(s) + O_2(g) \rightarrow 2SrO(s)$
$3Sr(s) + N_2(g) \rightarrow Sr_3N_2(s)$

Strontium forms both an oxide and a nitride.

- Barium (Ba)

$Ba(s) + O_2(g) \rightarrow BaO_2(s)$ (primarily peroxide)

$3Ba(s) + N_2(g) \rightarrow Ba_3N_2(s)$

Barium forms both an oxide (peroxide) and a nitride.

The metals from the list that form both an oxide and a nitride are lithium (Li), beryllium (Be), magnesium (Mg), strontium (Sr), and barium (Ba).

Question13

Identify the incorrect order against the property given in brackets

Options:

A.

$BeCO_3 < MgCO_3 < CaCO_3 < SrCO_3$ (Thermal stability)

B.

$BeSO_4 > MgSO_4 > CaSO_4 > SrSO_4$ (Solubility in water)

C.

$Li_2CO_3 > Na_2CO_3 > K_2CO_3 > Rb_2CO_3$ (Thermal stability)

D.

$BeCO_3 > MgCO_3 > CaCO_3 > SrCO_3$ (Solubility in water)

Answer: C

Solution:

(a) Thermal stability of group 2 carbonates : The thermal stability of carbonates of alkaline earth metals increases down the group as the cation's polarising power decreases. The order $BeCO_3 < MgCO_3 < CaCO_3 < SrCO_3$ is correct.

(b) Solubility of group 2 sulphates: The solubility of sulphates of alkaline earth metals decreases down the group. The order $BeSO_4 > MgSO_4 > CaSO_4 > SrSO_4$ is correct.

(c) Thermal stability of group 1 carbonates : The thermal stability of alkali metal carbonates increases down the group as the cation's polarising power decreases. The correct order is

$Li_2CO_3 < Na_2CO_3 < K_2CO_3 < Rb_2CO_3$

The given order,

$Li_2CO_3 > Na_2CO_3 > K_2CO_3 > Rb_2CO_3$,

is the reverse and therefore incorrect.

(d) Solubility of group 2 carbonates: The solubility of alkaline earth metal carbonates decreases down the group. The order $BeCO_3 > MgCO_3 > CaCO_3 > SrCO_3$ is correct.

Thus, the incorrect order is (c)..

Question14

Diborane on hydrolysis gives a compound X. The correct statement about X are

I. It is a tribasic acid

II. It is a weak monobasic acid

III. It has a layer structure

IV. It is highly soluble in water

Options:

A.

I and III

B.

II and III

C.

II and IV

D.

I and IV

Answer: B

Solution:

The hydrolysis of diborane (B_2H_6) gives boric acid (H_3BO_3) as the compound X. The reaction is

$B_2H_6 + 6H_2O \rightarrow 2H_3BO_3 + 6H_2$

The properties of boric acid are as follows.

- Acidity : Boric acid is a weak monobasic Lewis acid. It does not donate protons but rather accepts a hydroxide ion from water, making it a monobasic acid. Therefore,

statement II is correct, and statement I (tribasic acid) is incorrect.

- Structure : In the solid state, boric acid has a layered structure, where planar BO_3 units are linked by hydrogen bonds. This structure gives it a slippery feel. Therefore, statement III is correct.

- Solubility : Boric acid has low solubility in cold water, although its solubility increases with temperature. It is not considered highly soluble. Therefore, statement IV is incorrect.

Based on the analysis, the correct statements about compound X are II and III.

Question15

Choose the correct statement about allotropes of carbon.

I. Graphite has layered structure.

II. Buckminster fullerene is not aromatic in nature.

III. The distance between two adjacent layers in graphite is 141.5 pm .

IV. The hybridisation of carbon in graphite and Buckminster fullerene is same.

Options:

A.

I and IV

B.

I and II

C.

II and III

D.

III and IV

Answer: A

Solution:

Statement I : Graphite's structure is well-known to be layered, with sheets of hexagonal rings of carbon atoms stacked on top of one another. Thus, this statement is correct.

- Statement II : Buckminster fullerene (C_{60}) has a conjugated π-electron system spread over its spherical surface. This delocalisation of electrons gives it aromatic properties, making it an aromatic compound. Therefore, this statement is incorrect.
- Statement III The distance between the layers in graphite is approximately 335 pm . The value 141.5 pm represents the carbon-carbon bond length within a single layer, not the interlayer distance. Thus, this statement is incorrect.
- Statement IV In both graphite and buckminster fullerene, each carbon atom is bonded to three other carbon atoms, resulting in sp^2 hybridisation. The unhybridised p-orbitals form delocalised π-systems. Thus, this statement is correct.

The correct statements are I and IV.

Question16

Which of the following is a lung irritant that can lead to an acute respiratory disease in children?

Options:

A.

SO_2

B.

CO_2

C.

CO

D.

NO_2

Answer: D

Solution:

Nitrogen dioxide (NO_2) is a known air pollutant that acts as a strong lung irritant. When inhaled, it reacts with the moisture in the respiratory tract to form acids (HNO_2) and HNO_3) that can damage the delicate lung tissue. Exposure to NO_2 is particularly harmful to children, as it can increase their susceptibility to respiratory infections and lead to acute respiratory illnesses.

Let's examine the other options.

- Sulphure dioxide (SO_2) is also a potent lung irritant and a major cause of respiratory problems, especially in asthamtics. However, between the given options, NO_2 is often specifically cited for its role in causing acute respiratory disease.

- Carbon dioxide (CO_2) is a simple asphyxiant at high concentrations,

meaning it displaces oxygen. It is not a long irritant.

- Carbon monoxide (CO) is a toxic gas that impairs the blood's ability to carry oxygen by binding to hemoglobin. It is not a lung irritant, Therefore, nitrogen dioxide (NO_2) is the correct answer.

Question17

Arrange the following in decreasing order of their boiling points

(A) 2-methylbutane

(B) 2, 2-dimethyl propane

(C) Pentane

(D) Hexane

Options:

A.

$D > C > A > B$

B.

B > A > C > D

C.

$D > A > C > B$

D.

$B > C > A > D$

Answer: A

Solution:

The boiling point of alkanes is primarily determined by molecular mass and branching.

(a) Molecular mass : Hexane (C_6H_{14}) has a higher molecular mass than the other compounds (C_5H_{12}). Therefore, it has the highest boiling point.

- Order so far : $D > (A, B, C)$

(b) Branching : For the isomers of pentane (C_5H_{12}), boiling point decreases with increasing branching due to a reduction in surface area for intermolecular forces.

- Pentane (C) is a straight chain.

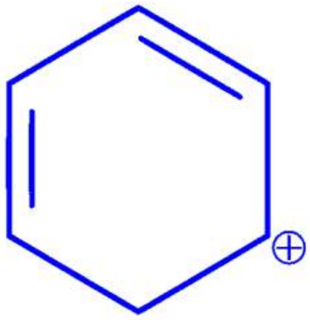
- 2-Methylbutane (A) has one branch.
- 2,2-Dimethylpropane (B) has two branches and is the most compact.
- Order for isomers : $C > A > B$. Combining both factors, the final decreasing order of boiling points is $D > C > A > B$.

Question18

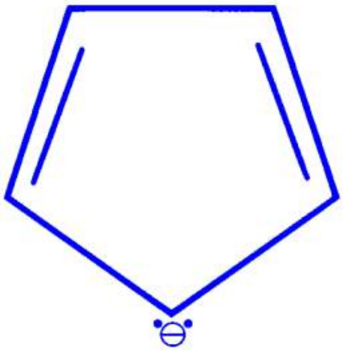
Which of the following is not an aromatic species?

Options:

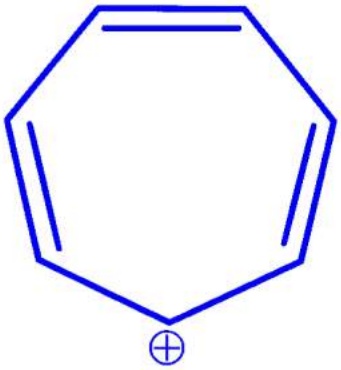
A.



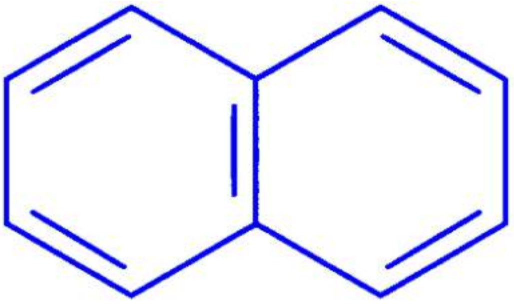
B.



C.



D.



Answer: A

Solution:

An aromatic species must be cyclic, planar, have a continuous ring of p-orbitals, and contain ($4n+2$) π-electrons (Huckel's rule). Let's apply these rules to each option

(c) Cycloheptatrienyl cation : It is cyclic, planar, and has 6π-electrons. This follows Huckel's rule aromatic.

(b) Cyclopentadienly anion: It is cyclic, planar, and has 6π-electrons. This follows Huckel's rule aromatic.

(a) Cyclohexadienyl cation: It is not aromatic. It is anti-aromatic as it has $4n$ electrons, which violates the Huckel's rule for aromaticity.

(d) Naphthalene : It is cyclic, planar, and has 10π-electrons. This follows Huckel's rule aromatic.

Therefore, the cyclohexadienyl cation is the only species that is not aromatic.

Question19

In the estimation of nitrogen by Kjeldahl's method 0.933 g of an organic compound ' X ' was analysed. Ammonia evolved was absorbed in 60 mL of 0.1 M H_2SO_4. The unreacted acid required 20 mL of 0.1 M NaOH for complete neutralisation. The compound ' X' is

Options:

A.

$C_6H_5CH_2NH_2$

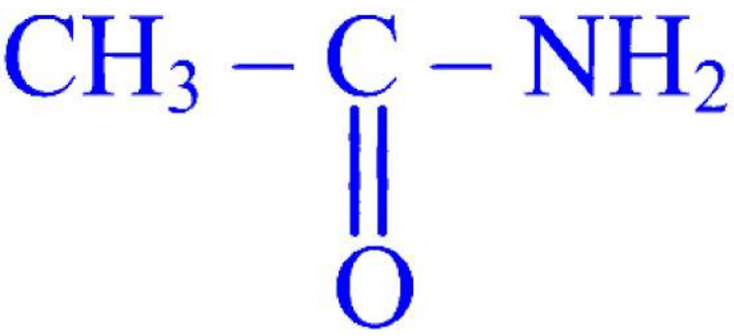
B.

$C_6H_5NH_2$

C.

$CH_3CH_2NH_2$

D.



Answer: B

Solution:

The organic compound X is $C_6H_5NH_2$ (aniline). Here's a concise breakdown of the solution.

We're using the Kjeldahl method to find the nitrogen content of compound X.

(A) Moles of acid reacted with ammonia

- Total H_2SO_4 added ; $0.060\ \mathrm{L} \times 0.1\mathrm{M} = 0.006$ mol.
- NaOH used in back-titration : $0.020\ \mathrm{l} \times 0.1\mathrm{M} = 0.002$ mol.
- Unreacted H_2SO_4 (from 2 NaOH : $1H_2SO_4$) : $0.002\ \mathrm{mol}/2 = 0.001$ mol.
- H_2SO_4 reacted with NH_3 : 0.006 mol 0.001 mol $= 0.005$ mol.

(B) Mass of nitrogen in compound X

- Moles of NH_3 (from $2NH_3$:

$1H_2SO_4$) : $0.005\ \mathrm{mol} \times 2 = 0.010$ mol.

- Mass of nitrogen ($\mathrm{N} = 14.01\ \mathrm{g/mol}$) : $0.010\ \mathrm{mol} \times 14.01\ \mathrm{g/mol} = 0.1401$ g.

(C) Percentage of nitrogen in compound X.

- Sample mass $= 0.933$ g
- $\%\ \mathrm{N} = (0.1401\ \mathrm{g}/0.933\ \mathrm{g}) \times 100\%$

$\approx 15.01\%$.

(D) Compare with options

- Aniline $(C_6H_5NH_2)$: Contains one N .

Molar mass $\approx 93\ \mathrm{g/mol}$. $\%\ \mathrm{N} = (14/93) \times 100\% \approx 15.05\%$.

- Other options (benzylamine, n-propylamine, acetamide) have significantly different nitrogen percentages (approx. 13.08\%, 23.73%, 23.73% respectively).
- Since compound X has approximately 15.01% nitrogen, it matches Aniline.

Question20

Which of the following is a least stable carbocation?

Options:

A.

$CH_3-\overset{\oplus}{C}H_2$

B.

$CH_2=\overset{\oplus}{C}H$

C.

$CH_2=CH\overset{\oplus}{-}CH_2$

D.

$C_6H_5-\overset{\oplus}{C}H_2$

Answer: B

Solution:

$CH_3-\overset{+}{C}H_2$ (Ethyl carbocation) :

This is a primary carbocation. It's stabilised by the $+I$ (inductive) effect of the methyl group and some hyperconjugation from the three alpha-hydrogens. The positive carbon is sp^2-hybridised.

(b) $CH_2=CH$ (Vinyl carbocation) : In this carbocation, the positive charge is on an sp hybridised carbon. An sp hybridised carbon is more electronegative than an sp^2 or sp^3 hybridised carbon, meaning it's less capable of accommodating a positive charge. The double bond itself cannot provide effective resonance stabilisation to this directly attached positive charge, and in fact, the proximity of the electron-withdrawing sp hybridised carbon makes it highly unstable. This is due to the electron-withdrawing nature of the double bond.

(c) $CH_2=CH-\overset{+}{C}H_2$ (Allyl carbocation) : This carbocation is highly stabilised by resonance. The positive charge is delocalised over two carbon atoms due to conjugation with the double bond.

(d) $C_6H_5-\overset{+}{C}H_2$ (Benzyl carbocation) This carbocation is also highly stabilised by resonance. The positive charge is delocalised into

the benzene ring due to conjugation.

- Comparing these, the vinyl carbocation $\left(CH_2=\overset{+}{C}H\right)$ is the least stable.

Question21

The incorrect statement about crystals with schottky defect is

Options:

A.

it is due to missing of equal number of cations and anions from lattice points.

B.

on the whole crystal is electrically neutral.

C.

it is shown by ionic compounds in which cation and anion are of almost same size.

D.

density of the crystal increases.

Answer: D

Solution:

The incorrect statement about crystals with a Schottky defect is : Density of the crystal increases. A Schottky defect involves an equal number of cations and anions missing from their lattice sites to maintain electrical neutrality. This defect is common in ionic compounds with similarly sized ions. Because ions are missing, the mass of the crystal decreases while the volume stays roughly the same, leading to a decrease in the crystal's density.

Question22

Two liquids ' A ' and ' B ' form an ideal solution. At 300 K , the vapour pressure of a solution containing 1 mole of ' A ' and 3 moles of ' B ' is 550 mm Hg . At the same temperature, if one more mole of ' B ' is added to the solution, the vapour pressure of solution increases to 560 mm Hg . Then the ratio of vapour pressures of A and B in their pure state is

Options:

A.

1 : 3

B.

3 : 1

C.

2 : 3

D.

3 : 2

Answer: C

Solution:

We use Raoult's Law,

$p_{\text{total}} = p_A^0 X_A + p_B^0 X_B$.

For the first solution (1 mole A, 3 moles B, total 4 moles), $p_{\text{total}} = 550$ mmHg.

$$550 = p_A^0(1/4) + p_B^0(3/4)$$
$$2200 = p_A^0 + 3p_B^0$$

For the second solution (1 mole A, 4 moles B , total 5 moles, after adding 1 more mole of B),

$p_{\text{total}} = 560$ mmHg.

$$560 = p_A^0(1/S) + p_B^0(4/S)$$
$$2800 = p_A^0 + 4p_B^0$$

Subtracting Equation 1 from Equation 2

$(p_A^0 + 4p_B^0) - (p_A^0 + 3p_B^0) = 2800 - 2200$

This simplifies to $p_B^0 = 600$ mmHg Substituting $p_B^0 = 600$ mmHg into equation 1

$$p_A^0 + 3(600) = 2200$$
$$p_A^0 + 1800 = 2200$$
$$p_A^0 = 400 \text{ mmHg}$$

Now, the ratio of p_A° and p_B° is $\frac{400}{600} = \frac{2}{3}$

Question23

The mole conductivity of acetic acid solution at infinite dilution is $390 \text{ S cm}^2 \text{ mol}^{-1}$. What is the molar conductivity of 0.01 M acetic acid solution (in $\text{Scm}^2 \text{ mol}^{-1}$)?

(Given $K_a\ (\text{CH}_3\text{COOH}) = 1.8 \times 10^{-5}$, assume $1 - \alpha = 1$)

Options:

A.

10.64

B.

16.54

C.

51.64

D.

15.64

Answer: B

Solution:

(a) Calculate the degree of dissociation (α).

For a weak acid, $K_a = C\alpha^2$ (assuming $1-\alpha \approx 1$).

Given, $K_a = 1.8 \times 10^{-5}$ and $C = 0.01\mathrm{M}$.

$\alpha^2 = K_a/C = \left(1.8 \times 10^{-5}\right)/0.01 = 1.8 \times 10^{-3}$
$\alpha = \sqrt{1.8 \times 10^{-3}} \approx 0.0424$

(b) Calculate the molar conductivity (Λ_m)

The relationship is $\Lambda_m = \alpha\Lambda_m^0$.

Given $\Lambda_m^\circ = 390\ \mathrm{S\ cm^2\ mol^{-1}}$.

$\Lambda_m = 0.0424 \times 390\ \mathrm{S\ cm^2\ mol^{-1}}$
$\approx 16.54\ \mathrm{S\ cm^2\ mol^{-1}}$.

Question24

The half-life of a zero order reaction $A \rightarrow$ products, is 0.5 hour. The initial concentration of A is $4\mathrm{molL}^{-1}$.

How much time (in hr) does it take for its concentration to come from $2.0\ \mathrm{mol\ L}^{-1}$ to $1.0\ \mathrm{mol\ L}^{-1}$?

Options:

A.

$1/4$

B.

$\frac{1}{8}$

C.

$1/2$

D.

$1/6$

Answer: A

Solution:

This problem involves a zero-order reaction, where the rate of reaction is independent of the reactant's concentration.

The integrated rate law for a zero-order reaction is given by $[A]_t = [A]_0 - kt$ where,

$[A]_t$ is the concentration at time t.
$[A]_0$ is the initial concentration.
k is the rate constant.

t is time.

The half-life ($t_{1/2}$) for a zero -order reaction is related to the initial concentration and rate constant by the expression

$t_{1/2} = \frac{[A]_0}{2k}$

Given

- $t_{1/2} = 0.5 \text{ h}$
- Initial concentration for half-life, $[A]_0 = 4 \text{ mol L}^{-1}$.

First, calculate the rate constant (k)

$$0.5\text{hr} = \frac{4 \text{ mol L}^{-1}}{2k}$$
$$k = \frac{4 \text{ mol L}^{-1}}{2 \times 0.5\text{hr}} = \frac{4 \text{ mol L}^{-1}}{1.0\text{hr}}$$
$$= 4 \text{ mol L}^{-1}\text{hr}^{-1}$$

Next, calculate the time (t) for the concentration to change from 2.0 mol L^{-1}

to 1.0 mol L^{-1};

Using the integrated rate law

$1.0 \text{ mol L}^{-1} = 2.0 \text{ mol L}^{-1}$

$-\left(4 \text{ mol L}^{-1}\text{hr}^{-1}\right)t$

Rearrange to solve for t

$\left(4 \text{ mol L}^{-1}\text{hr}^{-1}t = 2.0 \text{ mol L}^{-1} - 1.0 \text{ mol L}^{-1}\right)$

$4t = 1.0\text{hr}$

$t=\frac{1.0}{4}\text{hr}$

$t=1/4$ h or 0.25 h .

Question25

Match the following.

	List-I (Type of colloid)		List-II (Example)
A	Sol	I	Cloud
B	Foam	II	Whipped cream
C	Gel	III	Paint
D	Aerosol	IV	Butter

The correct answer is

Options:

A.

A-IV, B-II, C-III, D-I

B.

A-III, B-I, C-IV, D-II

C.

A-III, B-II, C-IV, D-I

D.

A-IV, B-I, C-II, D-III

Answer: C

Solution:

1. **Sol:** A colloidal system where a **solid is dispersed in a liquid**.
    - Examples: Paint, ink, blood, mud.
    - From List-II, **Paint** (III) is a solid dispersed in a liquid.
    - So, **A - III**.
2. **Foam:** A colloidal system where a **gas is dispersed in a liquid**.

- Examples: Whipped cream, soap lather, shaving cream.
- From List-II, **Whipped cream** (II) is gas (air) dispersed in cream (liquid).
- So, **B - II**.

3. **Gel:** A colloidal system where a **liquid is dispersed in a solid**. Gels are typically semi-solid.
   - Examples: Jelly, cheese, butter, gelatin.
   - From List-II, **Butter** (IV) is primarily water (liquid) dispersed in solid fat.
   - So, **C - IV**.
4. **Aerosol:** A colloidal system where a **solid or liquid is dispersed in a gas**.
   - Examples: Fog, mist, cloud (liquid in gas); smoke (solid in gas).
   - From List-II, **Cloud** (I) is fine droplets of water (liquid) dispersed in air (gas).
   - So, **D - I**.

Combining these matches:

- A - III
- B - II
- C - IV
- D - I

Question26

Observe the following statements

Statement I : The choice of reducing agent for the reduction of an oxide ore can be predicted by using Ellingham diagram, a plot of $\Delta G^{\ominus}$ vs T.

Statement II : According to Ellingham diagram, metal oxide with higher $\Delta G^{\ominus}$ is more stable than the oxide with lower $\Delta G^{\ominus}$.

The correct answer is

Options:

A.

Both Statement I and II are correct.

B.

Statement I is correct, but Statement II is not correct.

C.

Statement I is not correct, but Statement II is correct.

D.

Both Statement I and II are not correct.

Answer: B

Solution:

Statement I Analysis:

Ellingham diagrams plot the standard Gibbs free energy change ($\Delta G^{\ominus}$) for the formation of various metal oxides as a function of temperature. The principle driving the choice of reducing agent is based on the relative thermodynamic stability of the oxides. A metal can reduce the oxide of another metal if the $\Delta G^{\ominus}$ for the formation of its own oxide is more negative (i.e., its line is lower on the diagram) than the $\Delta G^{\ominus}$ for the formation of the oxide to be reduced, at a given temperature. This means that the overall reduction reaction will have a negative $\Delta G^{\ominus}$, making it thermodynamically feasible. Therefore, Ellingham diagrams are indeed used to predict the choice of reducing agent.

Statement I is correct.

Statement II Analysis:

The thermodynamic stability of a compound is inversely related to its standard Gibbs free energy of formation ($\Delta G^{\ominus}$). A more negative (lower) value of $\Delta G^{\ominus}$ for the formation of a compound indicates greater thermodynamic stability of that compound. Conversely, a higher (less negative or more positive) value of $\Delta G^{\ominus}$ indicates lower stability. Therefore, a metal oxide with a higher $\Delta G^{\ominus}$ is *less stable* than an oxide with a lower $\Delta G^{\ominus}$.

Statement II is incorrect.

Based on the analysis:

Statement I is correct.

Statement II is not correct.

This corresponds to Option B.

The final answer is $\boxed{B}$

Question27

Which one of the orders is correctly matched with the property mentioned against it?

Options:

A.

$H_2\,S < H_2O < H_2Se < H_2Te$ (Boiling point)

B.

$N_2O < NO < N_2O_3 < N_2O_4 < N_2O_5$ (Acidic nature)

C.

$HI < HCl < HBr < HF$ (Acidic nature)

D.

$H_2O < H_2S < H_2Se < H_2Te$ (Bond angle)

Answer: B

Solution:

Among the given orders, order given in options (b) i.e.

$N_2O < NO < N_2O_3 < N_2O_4 < N_2O_5$ (a

(Acidic Nature) is correct.

The correct order for remaining options are

(a) $H_2S < H_2Se < H_2Te < H_2O$

(Boiling point)

(c) $HF < HCl < HBr < HI$

(Acidic Nature)

(d) $H_2Te < H_2Se < H_2S < H_2O$

(Bond angle)

Question28

Noble gas ' X ' is used as a diluent for oxygen in modern diving apparatus and noble gas ' Y ' is used mainly to provide an inert atmosphere in high temperature metallurgical processes ' Y ' and ' X ' are respectively?

Options:

A.

He, Ar

B.

Ar, He

C.

He, Kr

D.

Ar, Kr

Answer: B

Solution:

Helium (X) is used as a diluent for oxygen in modern dividing apparatus and argon (Y) used to provide an inert atmosphere in high temperature

metallurgical process. Thus Y and X respectively are Ar, He.

Question29

The dibasic oxoacid of phosphorus on disproportionation given two products A and B. A and B are respectively.

Options:

A.

HPO_3, PH_3

B.

H_3PO_2, H_2O

C.

H_3PO_4, PH_3

D.

$H_4P_2O_6, H_3PO_2$

Answer: C

Solution:

Orthophosphorus acid (or phosphorous acid) on heating disproportionates to orthophosphoric acid (or phosphoric acid) and phosphine. The reaction involved is as follows.

$$4H_3PO_3 \rightarrow \underset{(A)}{3H_3PO_4} + \underset{(B)}{PH_3}$$

Thus, A and B are H_3PO_4 and PH_3 respectively.

Question30

The number of moles of oxalate ions oxidised by one mole of permanganate ions in acidic medium is

Options:

A.

2.5

B.

5.0

C.

1.5

D.

2.0

Answer: A

Solution:

The number of moles of oxalate ions oxidised by one mole of permanganate ions in acidic medium is 2.5. The complete reaction involved is as follows.

$$\frac{5}{2}C_2O_4^{2-} + MnO_4^- + 8H^+ \rightarrow Mn^{2+} + 4H_2O + 5CO_2$$

Question31

Total number of geometrical isomers possible for the complexes $[NiCl_4]^{2-}$, $[CoCl_2(NH_3)_4]^+$, $[Co(NH_3)_3(NO_2)_3]$ and $[Co(NH_3)_5Cl]^{2+}$ is

Options:

A.

2

B.

3

C.

4

D.

5

Answer: C

Solution:

The total number of geometrical isomers for the given complexes are

- $[NiCl_4]^{2-}$ - No geometrical isomer
- $[CoCl_2(NH_3)_4]^+$- octahedral complex of the type MA_4B_2. So can exist as cis and trans isomers. Thus 2 geometrical isomers are possible.
- $[Co(NH_3)_3(NO_2)_3]$ - Octahedral complex of the type MA_3B_3. So can exist as facial and meridional isomer. So, 2 geometrical isomers are possible.
- $[Co(NH_3)_5Cl]^{2+}$ - Octahedral complex of the type MA_5B. All positions are equivalent relative to the single unique ligand. So no geometrical isomer is possible.

Thus, total 4 geometrical isomers are possible.

Question32

Match the following.

List-I (Type of polymer)	List-II (Structure of the example)
(a) Fibre	I $-\!\!\left(CH_2-CH(Cl)\right)_n\!\!-$
(b) Elastomer	II $\left[NH-(CH_2)_6-N(H)-C(=O)-(CH_2)_4-C(=O)\right]_n$
(c) Thermosetting polymer	III $-\!\!\left(CH_2-C(Cl)=CH-CH_2\right)_n\!\!-$
(d) Thermoplastic polymer	IV $\left[NH-CO-NH-CH_2\right]_n$

The correct answer is

Options:

A.

A-II, B-IV, C-I, D-III

B.

A-II, B-III, C-IV, D-I

C.

A-III, B-I, C-IV, D-II

D.

A-III, B-II, C-IV, D-I

Answer: B

Solution:

The correct match is A-II, B-III, C-IV, D-I.

Question33

Maltose on hydrolysis gives two monosaccharide units. The incorrect statement about the monosaccharides formed is

Options:

A.

both are α-D glucose units only.

B.

one is $\alpha - D$ glucose and second one is β-D fructose.

C.

both are reducing sugars.

D.

in maltose, they are joined through 1,4-glycosidic linkage.

Answer: B

Solution:

The incorrect statement about monosaccharide units formed by hydrolysis of maltose is given in option (b). Its correct form is already given in option (a).

Question34

Identify the pair of drugs which act as antihistamines?

Options:

A.

Dimetapp, Seldane

B.

Iproniazid, Nardil

C.

Veonal, Valium

D.

Heroin, Codeine

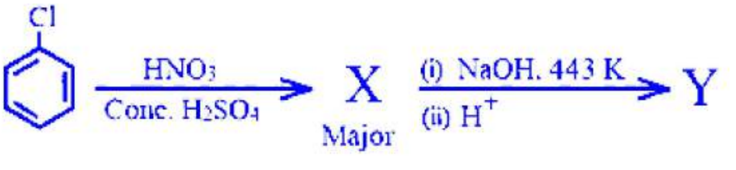
Answer: A

Solution:

Synthetic drugs like dimetapp (brompheniramine) and seldane (terfenadine) act as antihistamines. They interfere with the natural action of histamine by competing with histamine for binding sites of receptor where histamine exerts its effect.

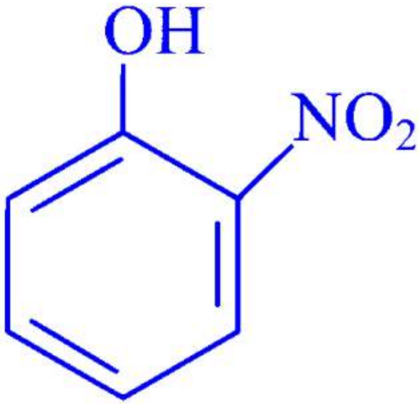
Question35

Identify the product ' Y ' in the given sequence of reactions



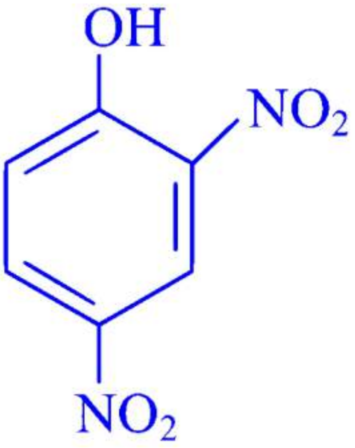
Options:

A.

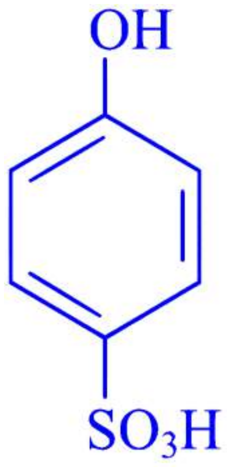


B.

C.



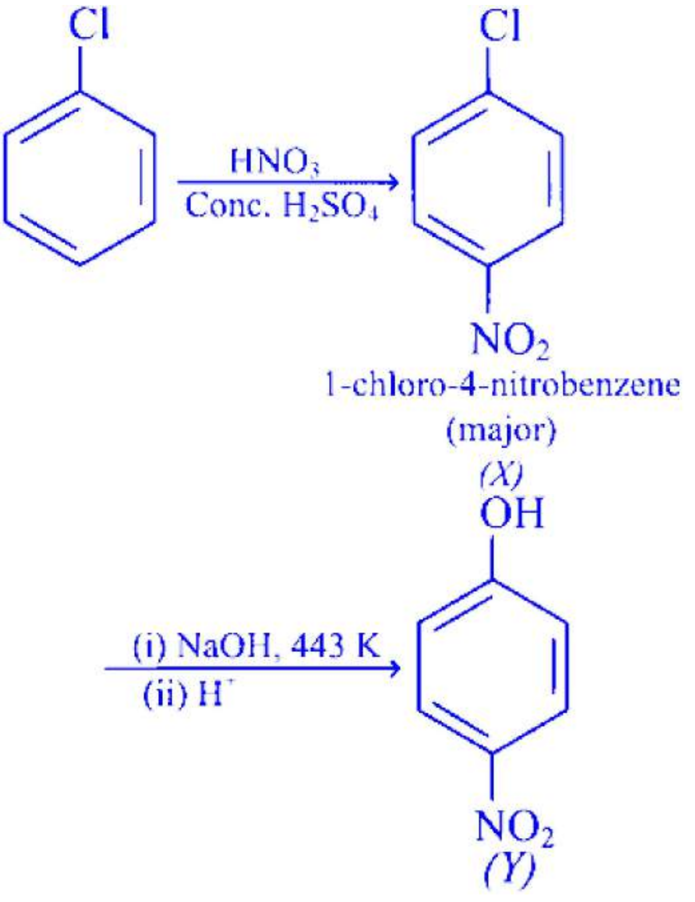
D.



Answer: B

Solution:

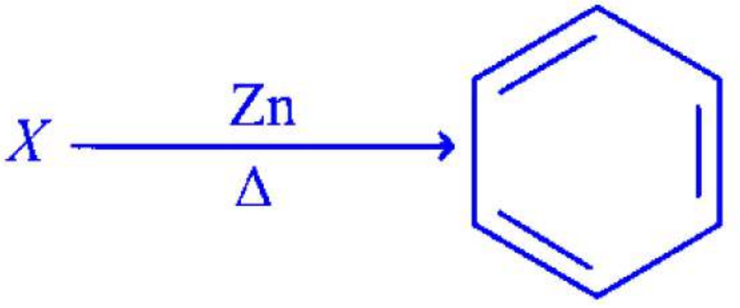
The complete reaction sequence is as follows.



Question36

What is ' Z ' in the given set of reactions?

$$C_6H_5OCH_3 \xrightarrow{HI} X + Y$$

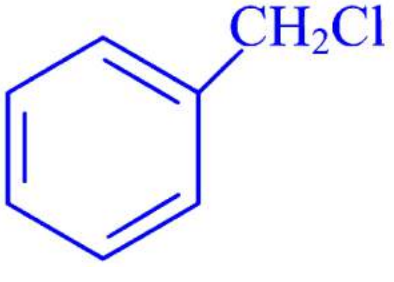


$$Y \xrightarrow[\text{Anhy. } AlCl_3]{C_6H_6} Z$$

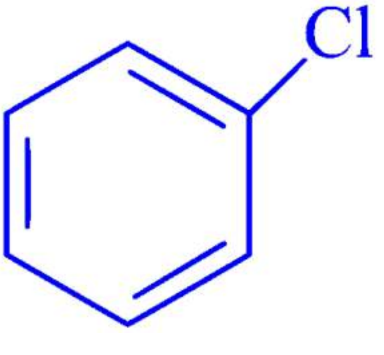
Options:

A.

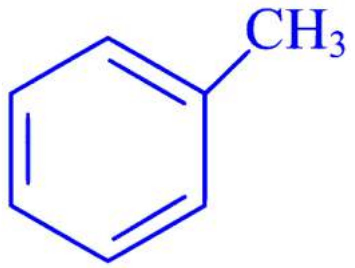
B.



C.



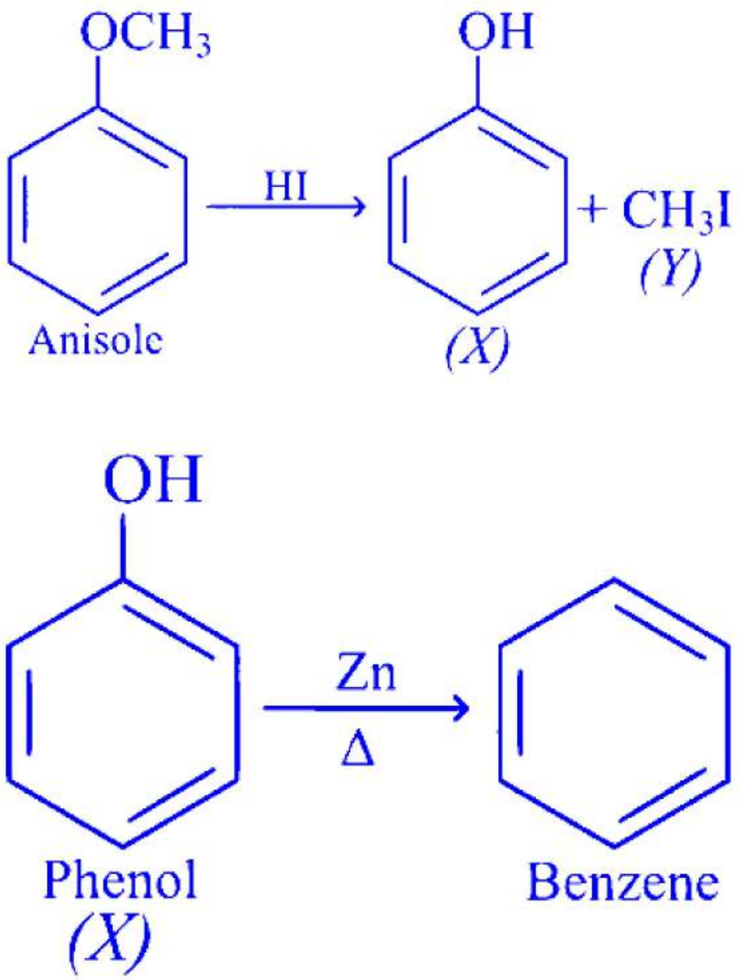
D.

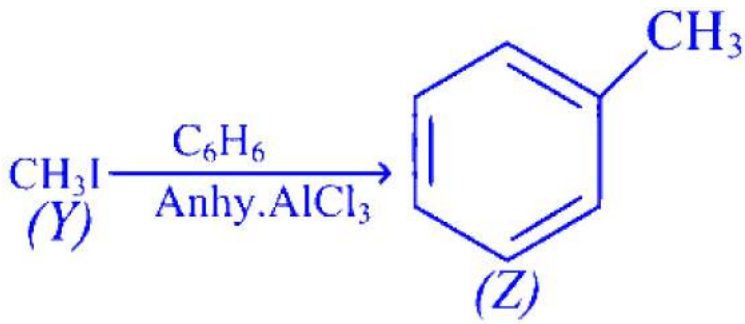


Answer: D

Solution:

The complete reaction sequence are as follows



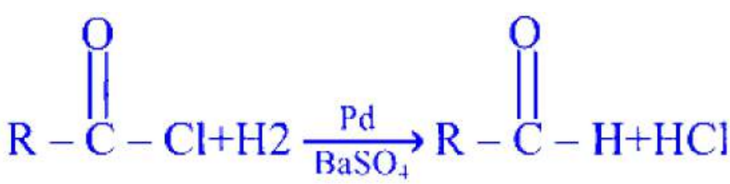


Question37

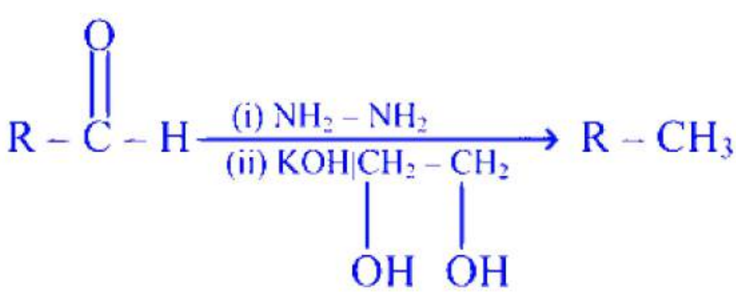
Which of the following reactions is an example of Clemmensen reduction?

Options:

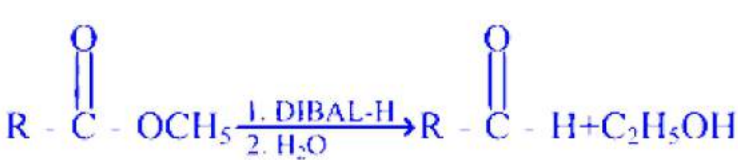
A.



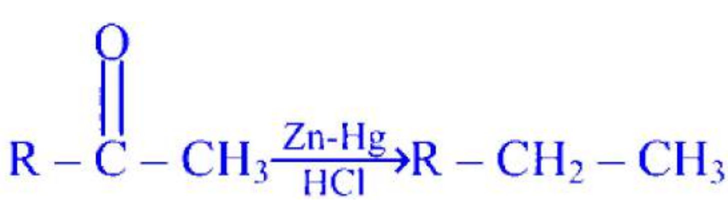
B.



C.



D.



Answer: D

Solution:

Example of Clemmensen reduction is given in option (d) i.e.

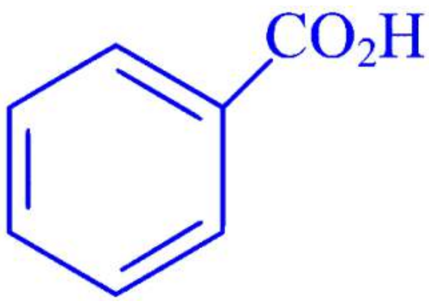
$$R-\overset{\overset{O}{\|}}{C}-CH_3 \xrightarrow[HCl]{Zn\text{-}Hg} R-CH_2-CH_3$$

Question38

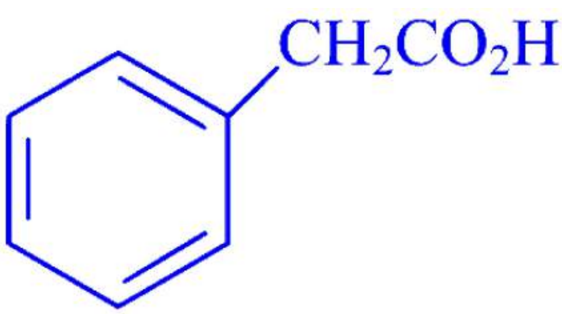
Which of the following can undergo Hell-Volhard-Zelinsky reaction?

Options:

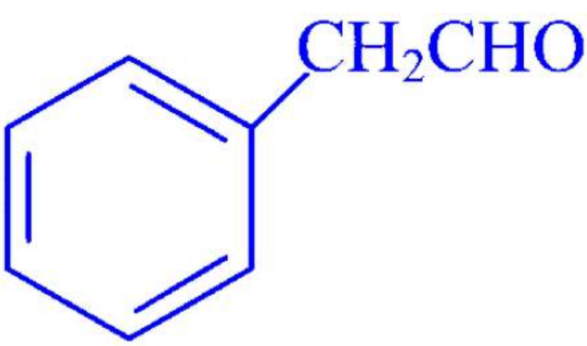
A.



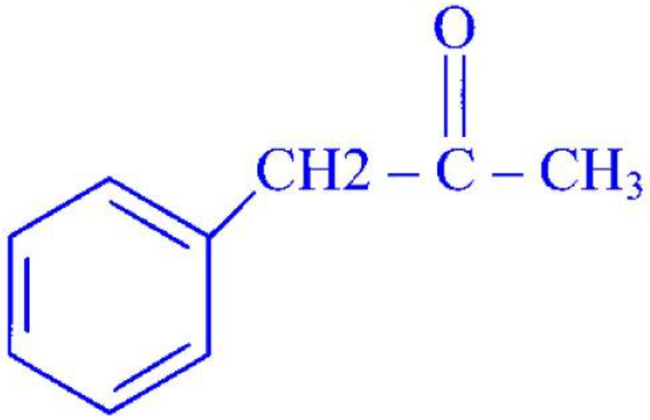
B.



C.



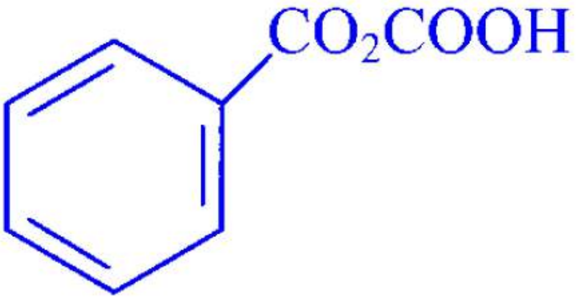
D.



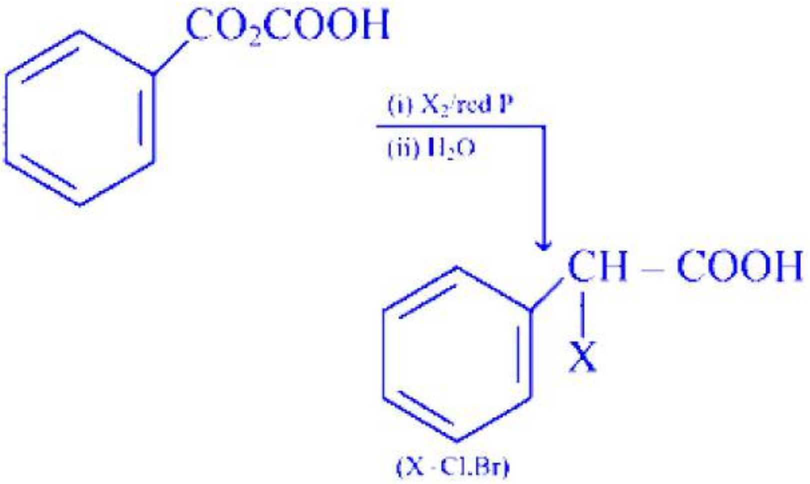
Answer: B

Solution:

Carboxylic acid having an α-hydrogen are halogenated at the α-position on treatment with chlorine or bromine in the presence of small amount of red phosphorus to give α-halocarboxylic acids. This reaction is called Hell-Volhard Zelinsky reaction. Thus among the given compounds,



will undergo this reaction is

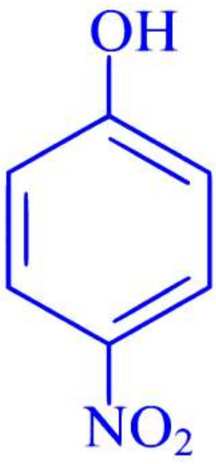


Question39

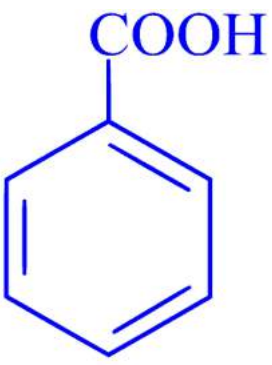
Which of the following has lowest $\mathrm{p}K_a$ value?

Options:

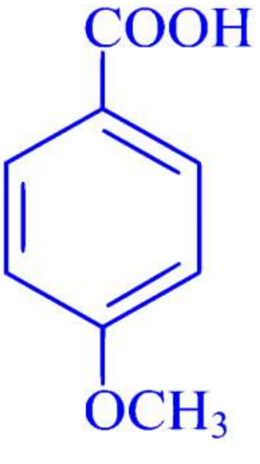
A.



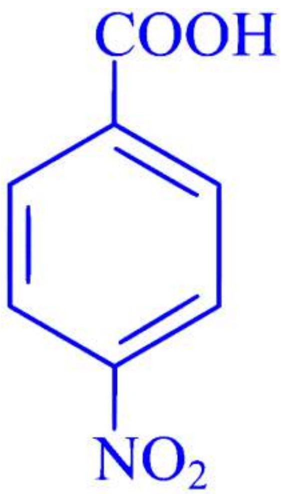
B.



C.



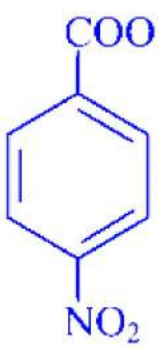
D.



Answer: D

Solution:

Smaller the pK_a value, stronger is the acid. Among the given compounds



is strongest acid thus it will have

lowest pK_a value.

Question40

The correct statement about the products B and

C in the given reactions are

$$\mathrm{CH_3CH_2OH} \xrightarrow[\text{Anhy. } \mathrm{ZnCl_2}]{\mathrm{HCl}} A \xrightarrow[\mathrm{AgCN}]{\text{Ethanolic}} \underset{\text{Minor}}{B} + \underset{\text{Major}}{C}$$

I. B and C are functional isomers.

II. With $\mathrm{H_2}/$ Catalyst B gives 1° amine and C gives 2° amine.

III. B on acid hydrolysis given formic acid and C gives $\mathrm{C_3H_6O_2}$.

IV. C forms isocyanate with HgO

Options:

A.

I and III

B.

II and III

C.

I, II and IV

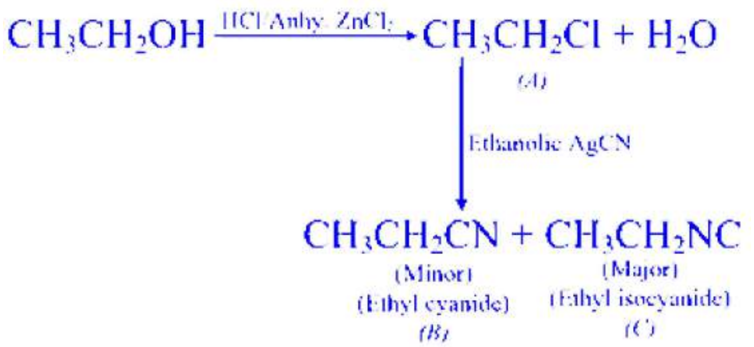
D.

II, III and IV

Answer: C

Solution:

The complete reaction is as follows



- Thus, B and C are functional isomers.
- With H_2 /catalyst B gives 1° amine and C gives 2° amine.
- C forms isocyanate with HgO . So, statement I, II and IV are correct.
- Ethyl cyanide on hydrolysis gives propionic acid while ethyl isocyanide on hydrolysis in acidic medium gives formic acid.

Physics

Question1

The range of weak nuclear force is of the order of

Options:

A.

10^{16} m

B.

10^{-10} m

C.

10^{10} m

D.

10^{-16} m

Answer: D

Solution:

The range of weak nuclear force is the order of 10^{-16} m. This force is responsible for radioactive decay and acte over very short distances within the nucleus.

Question2

A piece of length 3.532 m is cut from a rod of length 43.4 m . The length of the remaining rod in metre is (up to correct significant figures)

Options:

A.

39.9

B.

39.8

C.

39.868

D.

39.87

Answer: A

Solution:

To find the length of the remaining rod, we subtract the length of the cut piece from the original length of the rod.

Original length = 43.4 m

Cut length = 3.532 m

Remaining length = Original length - Cut length

Remaining length = 43.4 - 3.532

Let's perform the subtraction:

43.400 (adding zeros to align decimal places for calculation)

- 3.532

39.868

Now, we need to consider the rules for significant figures in subtraction. When adding or subtracting measurements, the result should be rounded to the same number of decimal places as the measurement with the fewest decimal places.

- The original length (43.4 m) has one decimal place.
- The cut length (3.532 m) has three decimal places.

The measurement with the fewest decimal places is 43.4 m, which has one decimal place. Therefore, our final answer must be rounded to one decimal place.

Our calculated exact result is 39.868.

To round 39.868 to one decimal place, we look at the second decimal place, which is 6. Since 6 is 5 or greater, we round up the first decimal place (8).

So, 39.868 rounded to one decimal place is 39.9.

Comparing this with the given options:

Option A: 39.9

Option B: 39.8

Option C: 39.868

Option D: 39.87

The correct answer is 39.9 m.

The final answer is $\boxed{39.9}$

Question3

A person wearing a parachute jumps off a plane from a height of 2 km from the ground and falls freely for 20 m before his parachute opens. After his parachute opens if he continues to move uniformly with the velocity attained due to his freefall, the total time taken by the person to reach the ground is (Acceleration due to gravity $= 10 \mathrm{~ms}^{-2}$)

Options:

A.

99 s

B.

100 s

C.

101 s

D.

102 s

Answer: C

Solution:

Step 1: Calculate the speed after falling 20 m

The person starts from rest, so the initial speed is 0. We use the formula:

$$v^2 = u^2 + 2gh$$
$$v = \sqrt{2 \times 10 \times 20}$$
$$v = \sqrt{400} = 20 \text{ m/s}$$

This means after falling 20 meters, the person is moving at 20 m/s.

Step 2: Find the time to fall the first 20 m

Use the formula for distance with constant acceleration from rest:

$$s = ut + \frac{1}{2}gt^2$$
$$20 = 0 + \frac{1}{2} \cdot 10 \cdot t^2$$
$$20 = 5t^2$$
$$t^2 = 4$$
$$t = 2 \text{ seconds}$$

So, it takes 2 seconds to fall the first 20 meters.

Step 3: Calculate the distance left after 20 m

The total height is 2000 meters. After falling 20 meters, the remaining distance is:

$$\text{Remaining distance} = 2000 - 20$$
$$= 1980 \text{ m}$$

Step 4: Find the time to travel the remaining 1980 m at constant speed

Now, the person moves at 20 m/s (from step 1). Time taken is:

$$t = \frac{1980}{20} = 99 \text{ seconds}$$

Step 5: Add both times to get the total time to reach the ground

Total time $= 2 + 99 = 101$ seconds.

Question4

A ball projected at an angle of 45° with the horizontal crosses two points at equal heights separated by a distance at times 2 s and 8 s respectively. The horizontal distance between the two points is

(Acceleration due to gravity $= 10 \text{ ms}^{-2}$)

Options:

A.

300 m

B.

400 m

C.

500 m

D.

600 m

Answer: A

Solution:

Time of flight of projectile

$(T) = 2 + 8 = 10 \text{ s}$

Horizontal distance between two points

$$
\begin{aligned}
&= u \cos 45^\circ \times 8 - u \cos 45^\circ \times 2 \\
&= 6u \cos 45^\circ
\end{aligned}
$$

$$T = \frac{2u \sin \theta}{g} = \frac{2u \sin 45^\circ}{10}$$

$$10 = \frac{2u \sin 45^\circ}{10}$$

$$u = 50\sqrt{2} \text{ m/s}$$

Horizontal distance between two points

$$= 6 \times 50\sqrt{2} \times \frac{1}{\sqrt{2}} = 300 \text{ m}$$

Question5

A truck of mass 8 ton is carrying a block of mass 2 ton. If a breaking force of 25 kN is applied on the truck, then the frictional force acting on the block is (Coefficient of static friction between the block and the truck is 0.3)

Options:

A.

6250 N

B.

6000 N

C.

5000 N

D.

1000 N

Answer: C

Solution:

Total mass of truck and block

$$M' = 8 + 2 = 10\text{ton} = 10 \times 1000 \text{ kg}$$
$$= 10^4 \text{ kg}$$

Acceleration produced due to application of 25 kN force,

$a = \frac{F}{M'} = \frac{25 \times 10^3}{10^4} = 2.5 \text{ m/s}^2$

Maximum frictional force acting on the block

$$F_{sm} = \mu_s mg$$
$$= 0.3 \times 2000 \times 10 = 6000 \text{ N}$$

The force required to decelerate the block with the same acceleration as the truck,

$F' = M_{\text{block}} \times a = 2000 \times 2.5 = 5000 \text{ N}$

Since, required force is less than the maximum static frictional force, thus, the block will not slide relative to the truck and the frictional force acting on the block will be equal to the required force to decelerate it with the truck's acceleration.

Thus, frictional force acting on the block $= 5000$ N

Question6

The work done in displacing a particle from $y = a$ to $y = 2a$ by a force $-\frac{K}{y^2}$ acting along Y-axis is

Options:

A.

$-\frac{5K}{8a}$

B.

$-\frac{14K}{8a^3}$

C.

$-\frac{K}{a^2}$

D.

$-\frac{K}{2a}$

Answer: D

Solution:

Work done

$$W = \int F dy = \int_a^{2a} \frac{-K}{y^2} dy = \left[\frac{K}{y}\right]_a^{2a}$$
$$= \left[\frac{K}{2a} - \frac{K}{a}\right] = \frac{-K}{2a}$$

Question7

Due to the presence of air resistance, if a body dropped from a height of 20 m reaches the ground with a speed of 18 ms^{-1}, then the time taken by the body to reach the ground is nearly

Options:

A.

1.8 s

B.

2.2 s

C.

2 s

D.

2.5 s

Answer: B

Solution:

Step 1: Find the acceleration

We know the final speed $v = 18 \text{ m/s}$, the starting speed $u = 0$, and the height $h = 20$ m.

Use the formula: $v^2 = u^2 + 2ah$

Put the values in: $18^2 = 0^2 + 2a \times 20$

This gives: $a = \frac{18\times18}{2\times20} = 8.1 \text{ m/s}^2$

Step 2: Find the time taken

Use the formula: $v = u + at$

Plug in the numbers: $18 = 0 + 8.1 \times t$

Solve for t:

$t = \frac{18}{8.1} = 2.22 \text{ s} \approx 2.2 \text{ s}$

Question8

A balance is made using a uniform metre scale of mass 100 g and two plates each of mass 200 g fixed at the two ends of the scale and the balance is pivoted at 45 cm mark of the scale. The error when 300 g weight is placed in the plate at 0 cm to weigh vegetables placed in the plate at 100 cm is

Options:

A.

36.4 g

B.

63.6 g

C.

200 g

D.

100 g

Answer: D

Solution:

Step 1: Understanding the Setup

We have a uniform meter scale that weighs 100 g. Each end of the scale has a plate that weighs 200 g. The scale is balanced on a support at the 45 cm mark. We put a 300 g weight on the plate at the 0 cm end and put vegetables (unknown weight x) on the plate at the 100 cm end.

Step 2: Calculating Clockwise Torque

The things making a clockwise turning effect (torque) around the pivot at 45 cm are:

- The 200 g plate at 100 cm, which is 55 cm from the pivot $(100 - 45 = 55)$,
- The vegetable weight (x) on this plate, also 55 cm from pivot,
- Part of the scale's own mass to the right of the pivot (100 g is the total mass, half of it is on each side, but for 5 cm past pivot, that's 5 g at 5 cm, but original calculation uses 100×5). Here we use 100 g at a distance of 5 cm $(50 - 45 = 5$ cm, but the calculation uses 5 cm for the rest mass).

So,

$$\text{Clockwise torque} = 200 \times 55 + x \times 55 + 100 \times 5$$
$$= 11,500 + 55x$$

Step 3: Calculating Anticlockwise Torque

The things making an anticlockwise torque are:

- The 200 g plate at 0 cm, 45 cm away from pivot,
- The 300 g weight at 0 cm, 45 cm away from pivot.

So, $$\text{Anticlockwise torque} = 200 \times 45 + 300 \times 45$$
$$= 9,000 + 13,500 = 22,500$$

Step 4: Setting Torques Equal (Balance)

For balance, clockwise torque = anticlockwise torque.

$$11,500 + 55x = 22,500$$
$$55x = 22,500 - 11,500 = 11,000$$
$$x = \frac{11,000}{55} = 200 \text{ g}$$

So, the actual mass of the vegetables is 200 g.

Step 5: Calculate the Error

The balance shows 300 g (apparent weight), but the real weight is 200 g (actual weight).

$\text{Error} = 300 - 200 = 100$ g

Question9

The ratio of radii of gyration of a thin circular ring and a circular disc of same radius about a tangential axis in their own planes is $\sqrt{12} : \sqrt{K}$. The value of K is

Options:

A.

10

B.

24

C.

5

D.

12

Answer: A

Solution:

Moment of inertia of ring about a tangential axis in its plane

$I=I_{\mathrm{CM}}+MR^2=\frac{MR^2}{2}+MR^2$

[The moment of inertia of a ring about its diameter $I_{\mathrm{CM}}=\frac{MR^2}{2}$]

$I=\frac{3}{2}MR^2$

If K_R be the radius of gyration, then

$MK_R^2=\frac{3}{2}MR^2 \Rightarrow K_R=\sqrt{\frac{3}{2}}R$

Moment of inertia of circular disc about tangential axis in its plane

$$\begin{aligned} I &= I_{CM}+MR^2 \\ &= \frac{MR^2}{4}+MR^2=\frac{5}{4}MR^2 \end{aligned}$$

[The moment of inertia of a disc about its diameter is $I_{\mathrm{CM}}=\frac{1}{4}MR^2$]

If K_C be the radius of gyration, then

$$MK_C^2=\frac{5}{4}MR^2$$

$$K_C=\sqrt{\frac{5}{4}}R$$

Since, $\frac{K_R}{K_C}=\frac{\sqrt{12}}{\sqrt{K}}$

$$\frac{\sqrt{\frac{3}{2}}R}{\sqrt{\frac{5}{4}}R}=\frac{\sqrt{12}}{\sqrt{K}} \Rightarrow \frac{3}{2}\times\frac{4}{5}=\frac{12}{K}$$

$$\Rightarrow \quad K=10$$

Question10

At a given place, to increase the number of oscillations made by a simple pendulum in one minute from 72 to 90 , the length of the pendulum is to be decreased by

Options:

A.

64%

B.

36%

C.

50%

D.

56%

Answer: B

Solution:

Step 1: Pendulum Frequency Formula

The time it takes for one swing (period) of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$, where l is length and g is gravity.

Frequency (f) is how many swings it makes in one second, so $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$.

Step 2: Relationship Between Frequency and Length

From this, frequency is inversely related to the square root of length: $f \propto \frac{1}{\sqrt{l}}$.

This means $\frac{f_2}{f_1} = \sqrt{\frac{l_1}{l_2}}$, where l_1 and f_1 are initial length and frequency, and l_2 and f_2 are final.

Step 3: Assign the Values

The number of swings in a minute is frequency per minute. Original frequency = 72, new frequency = 90.

So, $\frac{f_2}{f_1} = \frac{90}{72} = \frac{5}{4}$.

Now, $\frac{5}{4} = \sqrt{\frac{l_1}{l_2}}$.

Step 4: Find the New Length

Square both sides: $(\frac{5}{4})^2 = \frac{l_1}{l_2}$, so $\frac{25}{16} = \frac{l_1}{l_2}$.

Rearrange to get $l_2 = \frac{16}{25}l_1$.

Step 5: Calculate the Percentage Decreased

Decrease in length = $l_1 - l_2$.

Percentage decrease = $\frac{l_1 - l_2}{l_1} \times 100\%$.

Substituting values: $\frac{l_1 - \frac{16}{25}l_1}{l_1} \times 100\% = \frac{9}{25} \times 100\% = 36\%$.

Question11

If the orbital speed of a body revolving in a circular path near the surface of the Earth is $8 \mathrm{kms}^{-1}$, then the orbital speed of a body revolving around the Earth in a circular orbit at height of $19,200 \mathrm{~km}$ from the surface of Earth is (Radius of the Earth $=6400 \mathrm{~km}$)

Options:

A.

$4 \mathrm{kms}^{-1}$

B.

$6 \mathrm{kms}^{-1}$

C.

$7.5 \mathrm{kms}^{-1}$

D.

$9 \mathrm{kms}^{-1}$

Answer: A

Solution:

Orbital speed at radius r from Earth's centre

$v=\sqrt{\frac{G M}{r}}$

So, $\frac{v_2}{v_1}=\sqrt{\frac{r_1}{r_2}}$

where,

$$\begin{aligned} r_1 & =R=6400 \mathrm{~km} \\ r_2 & =R+h \\ & =6400+19200=25600 \mathrm{~km} \\ \frac{v_2}{8} & =\sqrt{\frac{6400}{25600}}=\sqrt{\frac{1}{4}}=\frac{1}{2} \\ v_2 & =8 \times \frac{1}{2}=4 \mathrm{~km} / \mathrm{s} \end{aligned}$$

Question12

The Young's modulus and Poisson's ratio of a material are respectively Y and σ. The force required to decrease the area of cross-section of a wire made of this material by $\triangle A$ is

Options:

A.

$\frac{Y\Delta A}{4\sigma}$

B.

$\frac{2Y\Delta A}{\sigma}$

C.

$\frac{Y\Delta A}{2\sigma}$

D.

$\frac{Y\Delta A}{\sigma}$

Answer: C

Solution:

$$A = \pi r^2$$
$$\therefore \frac{dA}{dr} = 2\pi r \Rightarrow dA = 2\pi r dr$$
$$\Delta A = 2\pi r \Delta r$$

Thus, $\frac{\Delta A}{A} = \frac{2\Delta r}{r}$ $\quad \ldots(i)$

∴ Poisson's ratio,

$$\sigma = -\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$
$$= \frac{-\Delta r/r}{\Delta l/l}$$

$$\Rightarrow \quad \frac{\Delta r}{r} = -\sigma\frac{\Delta l}{l} \quad \ldots(ii)$$
$$\therefore \quad Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta l/l}$$
$$\Rightarrow \quad \frac{\Delta l}{l} = \frac{F}{AY} \quad \ldots(iii)$$

∴ From Eq. (ii) and (iii), we have,

$\frac{\Delta r}{r} = \frac{-\sigma F}{AY}$ $\quad \ldots(iv)$

Now, from Eq. (i) and (ivq), we get

$$\frac{\Delta A}{A} = 2\left(-\frac{\sigma F}{AY}\right) \Rightarrow F = \frac{-Y\Delta A}{2\sigma}$$

$$\therefore |F| = \frac{Y\Delta A}{2\sigma}$$

Question13

A thin film of water is formed between two straight parallel wires each of length 8 cm separated by distance of 0.6 cm . The work done to increase the distance between the wires to 0.8 cm is (Surface tension of water $= 0.07 \mathrm{Nm}^{-1}$)

Options:

A.

33.6μ J

B.

22.4μ J

C.

11.2μ J

D.

44.8μ J

Answer: B

Solution:

When you pull the wires apart, the water film stretches, making its area bigger.

A thin film has two surfaces (top and bottom), so the increase in area is:

$\Delta A = 2 \times L \times \Delta d$

Let's define what we have:

Length of each wire, $L = 8\,\mathrm{cm} = 0.08\,\mathrm{m}$

Starting distance, $d_1 = 0.6\,\mathrm{cm} = 0.006\,\mathrm{m}$

Final distance, $d_2 = 0.8\,\mathrm{cm} = 0.008\,\mathrm{m}$

Change in distance:

$\Delta d = d_2 - d_1 = 0.008 - 0.006 = 0.002\,\mathrm{m}$

Calculate the change in area:

$\Delta A=2\times0.08\times0.002=3.2\times10^{-4}\,\mathrm{m}^2$

The work done (energy needed) is given by:

$W=T\Delta A$

Here, surface tension $T=0.07\,\mathrm{N\,m}^{-1}$

So,

$W=0.07\times3.2\times10^{-4}=0.224\times10^{-4}\,\mathrm{J}$

$W=22.4\times10^{-6}\,\mathrm{J}=22.4\,\mu\mathrm{J}$

Question14

A rain drop of diameter 1 mm falls with a terminal velocity of $0.7\ \mathrm{ms}^{-1}$ in air. If the coefficient of viscosity of air is 2×10^{-5} Pa-s, the viscous force on the rain drop is

Options:

A.

$132\times10^{-8}\,\mathrm{N}$

B.

$6.6\times10^{-8}\,\mathrm{N}$

C.

$26.4\times10^{-8}\,\mathrm{N}$

D.

$10.4\times10^{-8}\,\mathrm{N}$

Answer: A

Solution:

Viscous force

$$\begin{aligned}F&=6\pi\eta rv\\&=6\times3.14\times2\times10^{-5}\times0.5\times10^{-3}\times0.7\\&=13.2\times10^{-8}\,\mathrm{N}\end{aligned}$$

Question15

The temperature at which the reading on Fahrenheit scale becomes 90% more than the reading on Celsius scale is

Options:

A.

$280°F$

B.

$580°F$

C.

$608°F$

D.

$320°F$

Answer: C

Solution:

Step 1: Write what the problem says.

The Fahrenheit temperature (F) is 90% more than the Celsius temperature (C). This means:

$F = C + 0.9C = 1.9C$

Step 2: Use the formula between Celsius and Fahrenheit.

The usual formula connecting C and F is:

$\frac{C}{5} = \frac{F-32}{9}$

Step 3: Substitute what you found in Step 1 into the formula from Step 2.

Replace F with $1.9C$:

$\frac{C}{5} = \frac{1.9C-32}{9}$

Step 4: Solve for C.

Multiply both sides by 9 to get rid of the denominator:

$9C/5 = 1.9C - 32$

Now multiply both sides by 5:

$9C = 5(1.9C - 32)$

$9C = 9.5C - 160$

Take $9C$ from both sides:

$9.5C - 9C = 160$

$0.5C = 160$

So, $C = 320$

Step 5: Find F using the value of C.

Use $F = 1.9C$:

$F = 1.9 \times 320 = 608°\text{F}$

Question16

A rectangular ice box of total surface area of $1000\ \text{cm}^2$ initially contains 1.5 kg of ice at $0°\text{C}$. If the thickness of the walls of the box is 2 mm and the temperature outside the box is $42°\text{C}$, then the mass of the ice remaining in the box after 160 minutes is

(Thermal conductivity of the material of the box $= 10^{-2}\text{Wm}^{-1}\ \text{K}^{-1}$ and latent heat of the fusion of ice $= 336 \times 10^3\text{Jkg}^{-1}$)

Options:

A.

0.6 kg

B.

0.9 kg

C.

0.8 kg

D.

0.7 kg

Answer: B

Solution:

Use heat conduction,

$$Q=\frac{KA\Delta T\cdot t}{d}$$
$$Q=\frac{\left(10^{-2}\right)\times 0.1\times 42\times 9600}{2\times 10^{-3}}$$
$$Q=\frac{0.042\times 9600}{2\times 10^{-3}}=\frac{403.2}{2\times 10^{-3}}$$
$$=2.016\times 10^{5}\text{ J}$$

Melted ice,

$$Q=m_{\text{melted}}\cdot L\Rightarrow m=\frac{Q}{L}$$
$$m_{\text{melted}}=\frac{2.016\times 10^{5}}{336\times 10^{3}}=0.6\text{ kg}$$

remaining ice,

$$m_{\text{remaining}}=1.5-0.6=0.9\text{ kg}$$

Question17

At constant pressure, equal amounts of heat are supplied to a monoatomic gas and a diatomic gas separately. The ratio of the increases in internal energies of the two gases is

Options:

A.

1 : 1

B.

9 : 49

C.

3 : 7

D.

21 : 25

Answer: D

Solution:

For monoatomic gas, $C_V=\frac{3}{2}R$ and $C_p=\frac{5}{2}R$

For diatomic gas,

$$C_V=\frac{5}{2}R, C_p=\frac{7}{2}R$$
$$\Delta U=nC_V\Delta T \text{ and } \Delta Q=nC_p\Delta T$$
$$\therefore \quad \frac{\Delta U}{\Delta Q}=\frac{nC_V\Delta T}{nC_p\Delta T}=\frac{C_V}{C_p}=\frac{1}{\gamma}\Rightarrow\frac{\Delta U}{\Delta Q}=\frac{1}{\gamma}$$

For monoatomic gas, $\gamma_m=\frac{\frac{5}{2}R}{\frac{3}{2}R}=\frac{5}{3}$

For diatomic gas, $\gamma_d=\frac{\frac{7R}{2}}{\frac{5R}{2}}=\frac{7}{5}$

Given, $\Delta Q_d=\Delta Q_m=Q$

$$\therefore \quad \Delta U_m=\frac{Q}{\gamma_m}=\frac{Q}{5}=\frac{3}{5}Q$$
$$\Delta U_d=\frac{Q}{\gamma_d}=\frac{Q}{\frac{7}{5}}=\frac{5}{7}Q$$

$$\therefore \quad \frac{\Delta U_m}{\Delta U_d}=\frac{\frac{3}{5}Q}{\frac{5}{7}Q}=\frac{21}{25}$$
$$\therefore \Delta U_m:\Delta U_d=21:25$$

Question18

If the rms speed of the molecules of a gas at a temperature of $77^{\circ}\mathrm{C}$ is $50\ \mathrm{ms}^{-1}$, then the rms speed of the same gas molecules at a temperature of $150.5^{\circ}\mathrm{C}$ is

Options:

A.

$65\ \mathrm{ms}^{-1}$

B.

$35\ \mathrm{ms}^{-1}$

C.

$55\ \mathrm{ms}^{-1}$

D.

$45\ \mathrm{ms}^{-1}$

Answer: C

Solution:

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$V_{\text{rms}} \propto \sqrt{T}$$

$$\Rightarrow \frac{(V_{\text{rms}})_2}{(V_{\text{rms}})_1} = \sqrt{\frac{T_2}{T_1}}$$

$$= \sqrt{\frac{273+150.5}{273+77}} = \sqrt{\frac{423.5}{350}}$$

$$\Rightarrow (V_{\text{rms}})_2 = (V_{\text{rms}})_1 \times \sqrt{\frac{423.5}{350}}$$

$$= 50 \times \sqrt{1.21}$$

$$= 50 \times 1.1 = 55 \text{ m/s}$$

Question19

Two tuning forks of frequencies 320 Hz and 323 Hz are vibrated together. The time interval between a maximum sound and its adjacent minimum sound heard by an observer is

Options:

A.

$\frac{1}{6}$ s

B.

$\frac{1}{3}$ s

C.

$\frac{1}{12}$ s

D.

$\frac{1}{9}$ s

Answer: A

Solution:

Beat frequency

$f = |323 - 320| = 3$ Hz

$\therefore$ Beat period

$T = \frac{1}{f} = \frac{1}{3}$ s

The time interval between a maximum sound and its adjacent minimum sound is equal to half of the beat period.

i.e., Time interval $= \frac{T}{2} = \frac{\frac{1}{3}}{2} = \frac{1}{6}$ s

----

Question20

The frequency of sound heard by an observer moving towards a stationary source with certain speed is n_1 and if the observer moves away from the same source with same speed, the frequency of sound heard by the observer is n_2. If the speed of sound in air is $340 \mathrm{~ms}^{-1}$ and $n_1 : n_2 = 71 : 65$, then speed of observer is

Options:

A.

$36 \mathrm{~km/h}$

B.

$27 \mathrm{~km/h}$

C.

$15 \mathrm{~km/h}$

D.

$54 \mathrm{~km/h}$

Answer: D

Solution:

Step 1: Understanding the Doppler Effect for a Moving Observer

When the sound source is not moving and the observer moves toward it, the frequency they hear increases.

When the observer moves away from the stationary source, the frequency they hear decreases.

Step 2: Writing the Frequency Formulas

If the actual frequency of the source is n, the speed of sound is v, and the observer's speed is u:

When moving toward the source: $n_1 = n\left(\frac{v+u}{v}\right)$

When moving away from the source: $n_2 = n\left(\frac{v-u}{v}\right)$

Step 3: Using the Ratio Given in the Problem

We are told that the ratio of the two frequencies is $n_1 : n_2 = 71 : 65$. We set up an equation:

$\frac{n_1}{n_2} = \frac{v+u}{v-u} = \frac{71}{65}$

Step 4: Solving for Observer's Speed (u)

Cross-multiply the equation: $65(v+u) = 71(v-u)$

Expand: $65v + 65u = 71v - 71u$

Move both v terms to one side and u terms to the other: $65u + 71u = 71v - 65v$ $136u = 6v$

Divide both sides by 136: $u = \frac{6v}{136} = \frac{3v}{68}$

Step 5: Substitute Speed of Sound Value

The speed of sound is $v = 340 \text{ m/s}$.

Plug this value into the formula for u: $u = \frac{3\times340}{68} = \frac{1020}{68} = 15 \text{ m/s}$

To change 15 m/s into km/h, multiply by $\frac{18}{5}$: $15 \times \frac{18}{5} = 54 \text{ km/h}$

Question21

A cassegrain telescope uses two mirrors of radii of curvature 25 cm and 16 cm separated by a distance of 2.5 cm . The position of the final image of an object at infinity is

Options:

A.

40 cm from convex mirror

B.

4.44 cm from concave mirror

C.

4.44 cm from convex mirror

D.

40 cm from concave mirror

Answer: A

Solution:

Focal length of primary mirror

$f_1 = \frac{R_1}{2} = \frac{25}{2} = 12.5 \text{ cm}$

Since, object is at infinity, so the image formed by the primary mirror will be at its focus.

$\therefore \quad v_1 = f_1 = 12.5 \text{ cm}$

Focal length of secondary mirror,

$$f_2 = -\frac{R_2}{2} = \frac{-16}{2} = -8 \text{ cm}$$
$$\therefore \quad u = 12.5 - 2.5 = 10 \text{ cm}$$

Using mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$
$$\frac{1}{-8} = \frac{1}{v} + \frac{1}{-10} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{8} = -\frac{1}{40}$$
$$\Rightarrow \quad v = -40 \text{ cm}$$

Thus, the final image is formed at a distance of 40 cm from the convex mirror.

Question22

A convex lens of radii of curvature 6 cm and 12 cm is immersed in a liquid of refractive index 1.3. If the refractive index of the material of the lens is 1.5 , then the focal length of the lens when immersed in the liquid is

Options:

A.

39 cm

B.

13 cm

C.

26 cm

D.

52 cm

Answer: C

Solution:

$$\frac{1}{f} = \left(\frac{\mu_g}{\mu_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$= \left(\frac{1.5}{1.3} - 1\right)\left(\frac{1}{6} - \frac{1}{-12}\right)$$
$$\frac{1}{f} = \frac{0.2}{1.3} \times \frac{3}{12}$$
$$f = \frac{1.3 \times 12}{0.2 \times 3} = 13 \times 2 = 26 \text{ cm}$$

Question23

When unpolarised light from air incidents on the surface of a medium of refractive index $\sqrt{3}$, then the reflected light is totally polarised. The angle of refraction is

Options:

A.

30°

B.

53°

C.

60°

D.

37°

Answer: A

Solution:

Step 1: Using Brewster's Law

Brewster's law says that $\mu = \tan i_p$, where μ is the refractive index of the medium, and i_p is the polarising angle (angle of incidence for which the reflected light is completely polarised).

We know the refractive index $\mu = \sqrt{3}$. So, $\sqrt{3} = \tan i_p$.

We know that $\tan 60^\circ = \sqrt{3}$. So, $i_p = 60^\circ$.

Step 2: Finding the Angle of Refraction

When light is totally polarised after reflection, the angle between the reflected and refracted rays is 90°. So, $i_p + r = 90^\circ$, where r is the angle of refraction.

This means $r = 90^\circ - i_p$.

We already found $i_p = 60^\circ$, so $r = 90^\circ - 60^\circ = 30^\circ$.

Question24

An alpha particle and a proton are accelerated from rest in a uniform electric field. The ratio of the times taken by proton and alpha particle to attain equal displacements is

Options:

A.

$\sqrt{2} : 1$

B.

$1 : 2$

C.

$1 : \sqrt{2}$

D.

$2 : 1$

Answer: C

Solution:

Step 1: Displacement equations

The proton and the alpha particle both start from rest, so their displacement after some time is given by:

$s = \frac{1}{2}at^2$.

Since they travel the same distance:

$\frac{1}{2}a_\alpha t_\alpha^2 = \frac{1}{2}a_p t_p^2$

This means:

$a_\alpha t_\alpha^2 = a_p t_p^2$

Step 2: Acceleration in an electric field

The acceleration of a charged particle in an electric field E is:

$a = \frac{qE}{m}$

- For a proton, charge $= e$, mass $= m_p$

- For an alpha particle, charge $= 2e$, mass $= 4m_p$

So: - $a_p = \frac{eE}{m_p}$ - $a_\alpha = \frac{2eE}{4m_p} = \frac{eE}{2m_p}$

Step 3: Substitute accelerations into equation

Plug the accelerations into the earlier equation:

$$\left(\frac{eE}{2m_p}\right)t_\alpha^2 = \left(\frac{eE}{m_p}\right)t_p^2$$

Step 4: Solve for time ratio

Divide both sides by $\frac{eE}{m_p}$:

$$\frac{t_\alpha^2}{2} = t_p^2$$

This gives:

$$t_\alpha^2 = 2t_p^2$$

Taking square roots:

$$t_\alpha = \sqrt{2}t_p$$

So,

$$\frac{t_p}{t_\alpha} = \frac{1}{\sqrt{2}}$$

or

$$t_p : t_\alpha = 1 : \sqrt{2}$$

Question25

A parallel plate capacitor with air as dielectric has a capacitance of 4μ F. The space between the plates of the capacitor is completely filled with a material of dielectric constant 5 and charged to a potential of 100 V . The work done to completely remove the dielectric material after the capacitor is disconnected from the battery is

Options:

A.

0.1 J

B.

0.5 J

C.

0.6 J

D.

0.4 J

Answer: D

Solution:

Step 1: Find Initial Capacitance

The original capacitor has air and its capacitance is 4 μF. When a dielectric with constant 5 fills the gap, the new capacitance becomes 5 times bigger:

$C_i = 5 \times 4\ \mu\text{F} = 20\ \mu\text{F}$

Step 2: Find Charge on the Capacitor

The capacitor is charged to 100 V. The charge stored is:

$Q = C_i \times V = 20 \times 10^{-6}\ \text{F} \times 100\ \text{V} = 2 \times 10^{-3}\ \text{C}$

Step 3: Work Out Initial Energy With Dielectric

The energy stored to start with (with dielectric in place) is:

$U_i = \frac{Q^2}{2C_i} = \frac{(2\times10^{-3})^2}{2\times20\times10^{-6}} = \frac{4\times10^{-6}}{40\times10^{-6}} = 0.1\ \text{J}$

Step 4: Find Final Capacitance After Removing Dielectric

When the dielectric is taken out, the capacitance returns to air value: $C_f = 4\ \mu\text{F}$.

Step 5: Calculate Final Energy With Dielectric Gone

The charge on the plates stays the same because the capacitor is disconnected from the battery. The energy now is:

$U_f = \frac{Q^2}{2C_f} = \frac{(2\times10^{-3})^2}{2\times4\times10^{-6}} = \frac{4\times10^{-6}}{8\times10^{-6}} = 0.5\ \text{J}$

Step 6: Work Done In Removing the Dielectric

Work done is the increase in energy:

$W = U_f - U_i = 0.5 - 0.1 = 0.4\ \text{J}$

Question26

The potential difference between the terminals of a cell is 20 V when a current of 2 A flows through the circuit. When the direction of current in the circuit is reversed, the potential difference between the terminals of the cell is 30 V . The internal resistance of the cell is

Options:

A.

1Ω

B.

1.5Ω

C.

2Ω

D.

2.5Ω

Answer: D

Solution:

First Condition

When the current flows normally, the formula for the potential difference across the cell is: $V = E - ir$

We are given that $V = 20\,V$ and $i = 2\,A$. Plug these values into the formula:

$20 = E - 2r \quad ...(i)$

Second Condition

When the current direction is reversed, the formula for potential difference becomes: $V = E + ir$.

Here, $V = 30\,V$ and $i = 2\,A$, so:

$30 = E + 2r \quad ...(ii)$

Find the Internal Resistance

Now, subtract equation (i) from equation (ii):

$(30) - (20) = (E + 2r) - (E - 2r)$

$10 = 4r$

So, $r = \frac{10}{4} = 2.5\,\Omega$

Question27

A straight uniform wire of resistance 36Ω is bent in the form of a semi-circular loop. The effective resistance between the ends of the diameter of the semi-circular loop is

Options:

A.

$\frac{56}{9}\Omega$

B.

$\frac{36}{7}\Omega$

C.

$\frac{99}{7}\Omega$

D.

$\frac{77}{9}\Omega$

Answer: D

Solution:

Step 1: Total length of the wire

The total length of the wire is the length of the semicircle plus the diameter. That is:
$l = \pi R + 2R$

Step 2: Find resistance per unit length

Since the total resistance is 36 Ω for the entire wire, resistance per unit length is:
$\sigma = \frac{36}{\pi R + 2R}$

Step 3: Resistance along the diameter (*straight part*)

The length of the diameter is $2R$. So, resistance along the diameter is:
$R_1 = \sigma \times 2R = \frac{36}{\pi R + 2R} \times 2R = \frac{72}{\pi + 2}$

Step 4: Resistance along the semicircle

The length of the semicircle is πR. So, resistance along the semicircle is:
$R_2 = \sigma \times \pi R = \frac{36}{\pi R + 2R} \times \pi R = \frac{36\pi}{\pi + 2}$

Step 5: Connect the two paths in parallel

The diameter and semicircular parts connect at both ends, so they are in parallel. Use the formula for parallel resistances:
$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Step 6: Substitute the values for R_1 and R_2

$$R_{eq} = \frac{\frac{72}{\pi+2} \times \frac{36\pi}{\pi+2}}{\frac{72}{\pi+2} + \frac{36\pi}{\pi+2}} = \frac{72 \times 36\pi}{(\pi+2)(36\pi+72)}$$

Step 7: Simplify the expression further

$$= \frac{72 \times 36 \times 22 \times 7}{36 \times (36 \times 22 + 72 \times 7)} = \frac{72 \times 22 \times 7}{36(22+14)} = \frac{77}{9}\ \Omega$$

Question28

An alpha particle moving with certain speed towards east enters a uniform magnetic field directed vertically up. The alpha particle will then move in

Options:

A.

vertical circular path with the same speed.

B.

horizontal circular path with the same speed.

C.

vertical circular path with increased speed.

D.

vertical circular path with decreased speed.

Answer: B

Solution:

A magnetic Lorentz force will act on the α-particle in magnetic field, which provide necessary centripetal force to move an uniform circular path in magnetic field. According to Flemming's left hand rule, α-particle will move on horizontal circular path with the same speed.

Question29

The ratios of the voltage sensitivities, resistances and areas of the coils of two moving coil galvanometers A and B are $4:3, 3:4$ and $1:2$ respectively. If the number of turns of the coil of galvanometer A is 200 , then the number of turns of the coil of galvanometer B is (All other quantities remain same in both the cases)

Options:

A.

100

B.

150

C.

200

D.

400

Answer: A

Solution:

Voltage sensitivity

$$S_V = \frac{NAB}{kR}$$

$$\frac{S_{V_A}}{S_{V_B}} = \frac{N_A}{N_B} \cdot \frac{A_A}{A_B} \cdot \frac{R_B}{R_A}$$

$$\Rightarrow \quad \frac{4}{3} = \frac{200}{N_B} \times \frac{1}{2} \times \frac{4}{3} \Rightarrow N_B = 100$$

Question30

A solenoid of 1000 turns per metre has a core of material with relative permeability 400 . The windings of the solenoid are insulated from the core and a current of 2 A is passed through the solenoid. Then, the value of the magnetic intensity inside the solenoid is

Options:

A.

$2 \times 10^3 \mathrm{Am}^{-1}$

B.

$1.0 \mathrm{Am}^{-1}$

C.

$8 \times 10^5 \mathrm{Am}^{-1}$

D.

$794 \mathrm{Am}^{-1}$

Answer: A

Solution:

Magnetic intensity H does not depend on the type of material inside the solenoid.

It only depends on:

- The number of turns per metre (n)
- The current passing through the solenoid (I)

The formula for magnetic intensity is:

$H = nI$

Now, substitute the given values:

- $n = 1000$ turns per metre
- $I = 2\,\mathrm{A}$

So,

$$\begin{aligned} H &= 1000 \times 2 \\ &= 2000\ \mathrm{A/m} \\ &= 2 \times 10^3\ \mathrm{Am}^{-1} \end{aligned}$$

Question31

An emf of 2.8 mV is induced in a rectangular loop of area $150\ \mathrm{cm}^2$ when the current in the loop changes from 3 A to 8 A in a time of 0.2 s . Then, the self-inductance of the loop is

Options:

A.

$112\mu\mathrm{H}$

B.

$56\mu\mathrm{H}$

C.

$28\mu\mathrm{H}$

D.

$84\mu\mathrm{H}$

Answer: A

Solution:

$$\begin{aligned} |e| &= L\frac{dI}{dt} \\ 2.8 \times 10^{-3} &= L\frac{(8-3)}{0.2} \\ \Rightarrow \quad L &= \frac{2.8}{25} \times 10^{-3} = 0.112 \times 10^{-3}\mathrm{H} \\ &= 112 \times 10^{-6}\mathrm{H} = 112\mu\mathrm{H} \end{aligned}$$

Question32

A capacitor and a resistor of resistance $100\sqrt{3}\Omega$ are connected in series to an AC source of voltage $100\sin(200t)\mathrm{V}$, where ' t ' is time in second. If the phase difference between the voltage and the current in the circuit is 30°, then the capacitance of the capacitor is

Options:

A.

$30\mu\ \mathrm{F}$

B.

$50\mu\ \mathrm{F}$

C.

$100\mu\ \mathrm{F}$

D.

$150\mu\ \mathrm{F}$

Answer: B

Solution:

$$\begin{aligned}\tan\phi &= \frac{X_C}{R} = \frac{1}{\omega CR}\\ C &= \frac{1}{\omega R\tan\phi}\\ &= \frac{1}{200\times 100\sqrt{3}\tan 30^{\circ}}\\ &= \frac{1}{2\times 10^4} = 50\times 10^{-6}\ \mathrm{F} = 50\mu\ \mathrm{F}\end{aligned}$$

Question33

The amplitude of the electric field associated with a light beam of intensity $\frac{15}{\pi}\mathrm{Wm}^{-2}$ is

Options:

A.

$120 \mathrm{NC}^{-1}$

B.

$15 \mathrm{NC}^{-1}$

C.

$60 \mathrm{NC}^{-1}$

D.

$30 \mathrm{NC}^{-1}$

Answer: C

Solution:

Step 1: Write the formula connecting intensity and electric field.

The formula for the intensity (I) of a light beam in terms of the amplitude of the electric field (E_0) is:

$I = \frac{1}{2} \varepsilon_0 c E_0^2$

where ε_0 is the permittivity of free space, and c is the speed of light.

Step 2: Rearrange the formula to find E_0.

$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$

Step 3: Substitute the given values into the formula.

Given: $I = \frac{15}{\pi} \mathrm{~Wm}^{-2}$, $\varepsilon_0 = 8.85 \times 10^{-12} \mathrm{~C}^2 / \mathrm{N} \cdot \mathrm{m}^2$, $c = 3 \times 10^8 \mathrm{~m/s}$

$E_0 = \sqrt{\frac{2 \times \frac{15}{\pi}}{8.85 \times 10^{-12} \times 3 \times 10^8}}$

Step 4: Calculate the value.

When you solve the above, you get:

$E_0 = 60 \mathrm{~NC}^{-1}$

Question34

When photons incident on a photosensitive material of work function 1.5 eV , the maximum velocity of the emitted photoelectrons is $8 \times 10^5 \mathrm{~ms}^{-1}$. The stopping potential of the photoelectrons is

(Mass of the electron $= 9 \times 10^{-31}$ kg and charge of the electron $= 1.6 \times 10^{-19}$C)

Options:

A.

1.8 V

B.

1.5 V

C.

2.1 V

D.

2.4 V

Answer: A

Solution:

The photoelectric effect equation relates the energy of incident photons, the work function of the material, and the maximum kinetic energy of emitted photoelectrons:

$E_{photon} = \phi + KE_{max}$

The maximum kinetic energy (KE_{max}) of the emitted photoelectrons is related to their maximum velocity (v_{max}) by the formula:

$KE_{max} = \frac{1}{2}mv_{max}^2$

The stopping potential (V_s) is the potential difference required to stop the most energetic photoelectrons. The work done by the stopping potential must be equal to the maximum kinetic energy of the electrons:

$KE_{max} = eV_s$

We are given:

- Work function (ϕ) = 1.5 eV (Note: This is not directly needed to find the stopping potential if v_{max} is given, but it would be needed to find the photon energy.)
- Maximum velocity (v_{max}) = 8×10^5 ms^{-1}
- Mass of electron (m) = 9×10^{-31} kg
- Charge of electron (e) = 1.6×10^{-19} C

First, calculate the maximum kinetic energy (KE_{max}) using the given maximum velocity:

$KE_{max} = \frac{1}{2}mv_{max}^2$

$KE_{max} = \frac{1}{2} \times (9 \times 10^{-31}\text{ kg}) \times (8 \times 10^5\text{ ms}^{-1})^2$

$KE_{max} = \frac{1}{2} \times 9 \times 10^{-31} \times (64 \times 10^{10})$

$KE_{max} = \frac{1}{2} \times 9 \times 64 \times 10^{-31+10}$

$KE_{max} = 9 \times 32 \times 10^{-21}$

$KE_{max} = 288 \times 10^{-21}$ J

Now, use the relationship between maximum kinetic energy and stopping potential ($KE_{max} = eV_s$) to find the stopping potential (V_s):

$V_s = \frac{KE_{max}}{e}$

$V_s = \frac{288 \times 10^{-21} \text{ J}}{1.6 \times 10^{-19} \text{ C}}$

$V_s = \frac{288}{1.6} \times \frac{10^{-21}}{10^{-19}}$

$V_s = \frac{2880}{16} \times 10^{-2}$

$V_s = 180 \times 10^{-2}$

$V_s = 1.8$ V

The stopping potential of the photoelectrons is 1.8 V.

The final answer is $\boxed{1.8 \text{ V}}$.

Question35

The potential energy of an electron in an orbit of hydrogen atom is -6.8 eV . The de-Broglie wavelength of the electron in this orbit is

(r_o is Bohr radius)

Options:

A.

$2\pi r_0$

B.

$4\pi r_0$

C.

πr_0

D.

$3\pi r_0$

Answer: B

Solution:

Step 1: Find the Total Energy (TE) from the Given Potential Energy (PE)

The potential energy of the electron is given as -6.8 eV.

We use the relation: $\text{PE} = 2(\text{TE})$

This means: $\text{TE} = \frac{\text{PE}}{2} = \frac{-6.8}{2} = -3.4\,\text{eV}$

Step 2: Find the Value of n (Principal Quantum Number)

For a hydrogen atom, the total energy formula is: $\text{TE} = \frac{-13.6}{n^2}$

Set this equal to the TE we found: $-3.4 = \frac{-13.6}{n^2}$

Solve for n^2: $n^2 = 4$

So, $n = 2$

Step 3: The Electron is in the Second Orbit

Since $n = 2$, the electron is in the second Bohr orbit.

Step 4: Use Bohr's Model and de-Broglie Hypothesis

According to Bohr and de-Broglie:

$2\pi r_n = n\lambda$

This gives: $\lambda = \frac{2\pi r_n}{n}$

Step 5: Use the Formula for the Radius of Orbit

Radius of the nth orbit: $r_n = n^2 r_0$

For $n = 2$ (second orbit): $r_2 = 2^2 r_0 = 4r_0$

Step 6: Calculate the de-Broglie Wavelength

Plug r_2 into the earlier formula:

$\lambda = \frac{2\pi r_2}{2} = \frac{2\pi}{2} \times 4r_0 = 4\pi r_0$

Question36

If a radioactive substance decays 10% in every 16 hours, then the percentage of the radioactive substance that remains after 2 days is

Options:

A.

82.2

B.

18.8

C.

27.1

D.

72.9

Answer: D

Solution:

Total time, $t = 2$ days

$= 2 \times 24$ hours
$= 48$ hours

Number of intervals of 16 hours $= \frac{48}{16} = 3$

Decays per interval = 10\%

Remaining fraction $= 0.9$

Remaining fraction after 3 intervals

$$f^n = (0.9)^3$$
$$= 0.729 = 72.9\%$$

Question37

If a nucleus P converts into a nucleus Q by the decay of one alpha particle and two β^-particles, then the nuclei P and Q are

Options:

A.

isotopes

B.

isobars

C.

isotones

D.

isomers

Answer: A

Solution:

Let X and Y be the mass number and atomic number of nucleus P.

$\therefore {}_{Y}^{X}P \longrightarrow Q + \alpha + 2\beta^{-}$

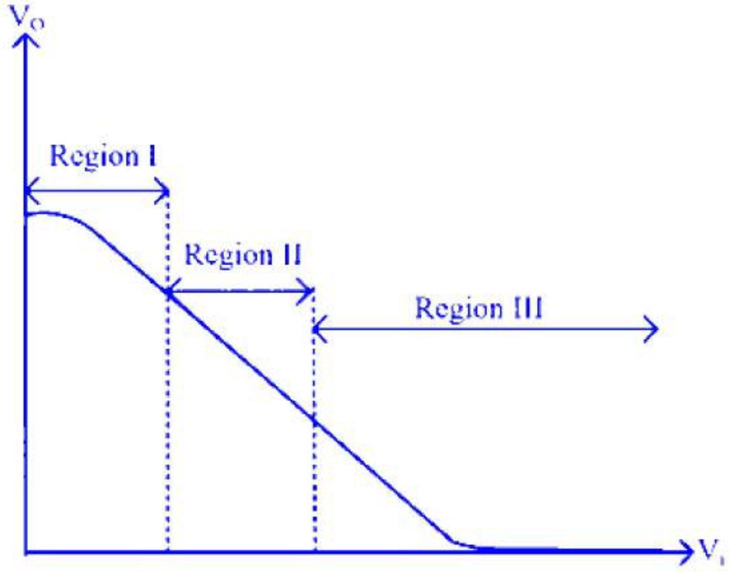
$\therefore$ Mass number of $Q = X - 4$

Atomic number of $Q = Y - 2 + 2 = Y$

Thus, we see that nuclei P and Q have same atomic number but different mass number. Thus, P and Q are isotopes.

Question38

The graph between the input voltage (V_i) and the output voltage (V_o) of a transistor connected in common emitter configuration is shown in the figure. The active, saturation and cutoff regions of the transistor are respectively



Options:

A.

I, II and III

B.

II, III and I

C.

I, III and II

D.

III. I and II

Answer: B

Solution:

From the graph

In region I, output voltage is high as V_0 when the input voltage V_i is low. i.e., transistor is off. This is cutoff region.

In region II, output voltage decreases as input voltage increases. This is the

situation when transistor operates as an amplifier.

Thus, in region II, transistor is in active mode.

In region III, output voltage is low when input voltage is high i.e., maximum current flow through the collector. Thus, transistor is in saturation region.

Question39

Which of the following logic gates is a universal gate?

Options:

A.

AND

B.

OR

C.

NOT

D.

NAND

Answer: D

Solution:

NAND and NOR gate are knows as universal gate because any logic gate can be implemented using these two gates.

Question40

The layer of the atmosphere which efficiently reflects high frequency waves particularly at night is

Options:

A.

troposphere

B.

stratosphere

C.

mesosphere

D.

thermosphere

Answer: D

Solution:

The correct answer is **D) thermosphere**.

Here's why:

1. **Ionosphere:** The reflection of radio waves is primarily the function of the **ionosphere**. The ionosphere is not a distinct layer in the same way as the troposphere or stratosphere, but rather a region within the upper atmosphere (primarily within the **thermosphere** and extending into the mesosphere) where gases are ionized by solar radiation.
2. **Layers of the Ionosphere:** The ionosphere consists of several ionized layers: D, E, F1, and F2.
   - **D-layer:** This is the lowest layer. It absorbs high-frequency (HF) waves during the day and largely disappears at night.
   - **E-layer:** This layer reflects some HF waves during the day but weakens considerably at night.
   - **F-layer (F1 and F2 merging into F-layer at night):** These are the highest and most intensely ionized layers. During the day, they are distinct F1 and F2 layers. **At night, the D and E layers largely dissipate due to the absence of solar radiation, and the F1 and F2 layers merge into a single, strong F-layer.** This strong F-layer at night is highly efficient at reflecting high-frequency (HF) radio waves back to Earth, enabling long-distance radio communication (skywave propagation).
3. **"Particularly at night"**: This phrase is key. At night, the lower D and E layers fade, reducing absorption and allowing HF waves to reach the higher F-layer of the ionosphere (within the thermosphere), where they are reflected much more effectively.

Therefore, the thermosphere, which contains the F-layer of the ionosphere, is responsible for efficiently reflecting high-frequency waves, particularly at night.